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The foundations of Chemical Reaction Network Theory, which aims at the investigation of the mathematical models of chemical and biological systems, was developed by Feinberg, Horn, and Jackson in the 1970’s. The purpose of this thesis is to revisit and improve the deficiency-oriented theory of mass action systems. The classical Deficiency-Zero- and Deficiency-One Theorems answer questions about the existence, uniqueness, and stability properties of the positive steady states. The main goal of this thesis is to investigate the existence of positive steady states of such deficiency-one mass action systems for which this question is not answered by the Deficiency-One Theorem.

The Deficiency-One Theorem states that there exists a unique positive steady state in each positive stoichiometric class for every weakly reversible deficiency-one mass action systems with single linkage class (regardless of the values of the rate coefficients). Under the extra assumption that the set of positive steady states is nonempty, the previous statement remains valid even if we omit the weak reversibility. However, the question of the non-emptiness of the set of positive steady states in the non weakly reversible case has not been addressed so far. In this thesis, we show that a trivially obtained necessary condition also serves as a sufficient condition to the non-emptiness of the set of positive steady states. Thus, we make the Deficiency-One Theorem complete in respect of the existence of the positive steady states. The obtained equivalent condition involves the rate coefficients. This raises the natural question of whether we can characterise those reaction networks for which the associated mass action system has nonempty set of positive steady states for any choice of rate coefficients. Using more involved graph theoretical arguments, we provide such a characterisation.

As a generalisation of the Deficiency-One Theorem to another direction, we prove the existence of a positive steady state in each positive stoichiometric class for every weakly reversible deficiency-one mass action systems with multiple linkage classes (regardless of the values of the rate coefficients). It turned out that independently of our work, Deng, Feinberg, Jones, and Nachman obtained the same conclusion without any assumption on the deficiency. Thus, their (yet unpublished) result is substantially more general. The proof by Deng et al. owes more to geometric ideas while our work uses more strictly algebraic methods. Most of the intermediate results of Deng et al. rely heavily on the weak reversibility of the network, while in our reasoning the weak reversibility of the network becomes crucial only in the concluding steps. Rather, we take advantage of the fact that the deficiency of the network is assumed to be one.