1.1 Exercise. Let $G_1$ and $G_2$ be two simple graphs with $\Delta(G_1, G_2) = 0$. Prove that there is a simple graph $G$ and $n_1, n_2 \geq 1$ such that $G_1 \cong G(n_1)$ and $G_2 \cong G(n_2)$.

1.2 Exercise. For a graphon $W$, define the following quantities analogous to the minimum degree and maximum degree of a graph:

\[ \delta(W) = \min_{x \in [0,1]} \int_0^1 W(x,y) \, dy, \quad \Delta(W) = \max_{x \in [0,1]} \int_0^1 W(x,y) \, dy, \]

Prove that for any tree $T$ on $k$ nodes, $\delta(W)^k \leq t(T, W) \leq \Delta(W)^k$.

1.3 Exercise. We define a (random) graph sequence $G_n$ as follows. We start with $G_1$ consisting of a single node. For $n \geq 2$, we obtain $G_n$ from $G_{n-1}$ by creating a new node, connecting it to node $i$ with probability $1 - i/n$ for $1 \leq i \leq n - 1$, and connecting every pair of nonadjacent nodes with probability $2/n$ (all these probabilistic decisions are made independently). Prove that the sequence $G_n$ tends to the graphon $W(x,y) = 1 - xy$ with probability 1.

1.4 Exercise. Starting with a single node, at each step we create a new node, flip a coin, and connect the new node either to all previous nodes, or to none of them, depending on the outcome of the coin flip. Prove that with probability 1, the sequence of graphs we obtain converges to the graphon

\[ W(x,y) = \begin{cases} 1, & \text{if } x + y \leq 1, \\ 0, & \text{otherwise}. \end{cases} \]

1.5 Exercise. (a) Let $K'_r$ denote the graph obtained by deleting an edge from the complete graph $K_r$ on $r$ nodes. Prove that for every graphon $W$,

\[ t(K'_r+1, W) \geq \frac{t(K_r, W)^2}{t(K_{r-1}, W)}. \]

(b) Prove that

\[ t(K_{r+1}, W) - t(K_r, W) \leq r(t(K_{r+1}, W) - t(K'_r+1, W)). \]

(c) Prove that

\[ r \frac{t(K_r, W)}{t(K_{r-1}, W)} \leq (r - 1) \frac{t(K_{r+1}, W)}{t(K_r, W)} + 1. \]