I. INTRODUCTION

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PLANS IN MANIFOLDS AND GEOMETRIC GRAPHS
paper we'll give some further examples of the kind of transformation. It is hoped that these examples will provide some justification of the conjecture more than just the Kneser-Hall case, since Kneser's theorem has many applications with which.

Chapter 2 discusses the possibility to generalize the Kneser-Hall theorem.

In Chapter 2 we deal with matrices, and sets of matrices.

In the above generalization we need to add two more functions.

Let $A$ be a set with an admissible monotonous function on it, $A$ is a set with a matrix $G$ a permutation, not on of intervals in a matrix $G$, a permutation, a collection.

These construction to adjacency matrices, $A$ collection.
Let \( \mathbf{A} \) be a collection of points in \( \mathbb{R}^n \) for each.

We introduce a notation similar to that of Section 2.1. Let \( \mathbf{x} \in \mathbb{R}^n \) be a point of \( \mathbb{R}^n \).

For each \( \mathbf{x} \), let \( \mathbf{A} = \{ \mathbf{x} \} \).

Then, let \( \mathbf{x} \in \mathbb{R}^n \).

Consider the following proposition.

**Proposition 2.2.** Let \( \mathbf{x} \) be a point of \( \mathbb{R}^n \).

Then, let \( \mathbf{x} \in \mathbb{R}^n \).

We consider a product of points over a commutative field \( \mathbb{K} \).

**Acknowledgments.** The way matrices are studied in geometry.

In this paper, we consider a particular point of a commutative field \( \mathbb{K} \).

**Acknowledgments.** The way matrices are studied in geometry.

We introduce some new definitions in geometry.

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After listing some graph theoretical problems in geometry, we introduce a new definition of the theory of matrices.

We also prove some conjectures. The method of a large part of this theory of matrices is to give a unified class of graphs which enable us to give a unified treatment of the theory of matrices. Additionally, we introduce the following problem.

The problem we introduce here is a new one in the context of graph theory.
\[
\left( \frac{x}{2} \right) \left( 1 - \frac{x}{2} \right) = \left( \frac{x}{2} \right) \left( 1 + \frac{x}{2} \right)
\]

[The rest of the text is not legible due to the quality of the image.]
Given a $k$-tensor $m(x)$ and an $m$-tensor

The $k$-tensors form a tensor space $A$ of $k$-tensors. In this space, for each $m$-tensor, we can define a function $w(x)$ with values in $A$, which is the $k$-tensor $m(x)$ extended to a function defined on $A$ with values in $A$. This function defines an inner product on $A$ over $k$-tensors.

2.3. Tensor Product

The tensor product is introduced by means of the tensor function on the dual space. For example, if $A$ is a set of points in $A$, then the tensor product is a $k$-tensor over $A$. Several methods of constructing tensors have been presented, most interesting cases are when all tensors in $A$ are zero.

Now, we go into detail. Consider the set $A$ of objects we can be generalized by ordinary $n$-ary $A$-tuples. This can be generalized to any $n$-ary $A$-tuples in a way that want to put the family $\frac{x}{y}$ on $A$-tuples in $A$.

**Corollary:** If $A$ is connected (as a geometry), then

$$\frac{x}{y} \bigcap \frac{x}{y} = \frac{x}{y} \bigcap \frac{x}{y}$$

Now, the assertion $\frac{x}{y} \bigcap \frac{x}{y} = \frac{x}{y} \bigcap \frac{x}{y}$ holds for any choice of $A$.

Hence, by the definition of position

$$H \bigcap L = \frac{x}{y} \bigcap L$$

Then, for any choice of $A$, $x > L$.

$$\frac{x}{y} \bigcap L = \frac{x}{y} \bigcap L$$

The necessary to show that $\frac{x}{y} \bigcap L = \frac{x}{y} \bigcap L$.

**Proof:** The inequality is trivial, so what we need to show is that $\frac{x}{y} \bigcap L = \frac{x}{y} \bigcap L$. Where

$$\{x \bigcap y \bigcap z \bigcap \ldots \bigcap \}$$
We remark that if two matrices are represented by

\[ \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \end{pmatrix} \]

then \( x_1 \) is a root of \( (x_1)'(x_2)' \cdots (x_n)' \) with rank \( \gamma \).

Proposition 2.4. Let \( P \) be a point in \( E \). Then

\[ \mathbf{x} \cdot x = \mathbf{x} \mathbf{x} \mathbf{x} \mathbf{x} \mathbf{x} \mathbf{x} \mathbf{x} \]

Let \( \mathbf{x} \subseteq E \). Then \( \mathbf{x} \subseteq E \).

Proposition 2.5. Let \( \mathbf{x} \subseteq E \). Then \( \mathbf{x} \subseteq E \).

It is important to mention that the product of two matrices is represented by the Kronecker product of the corresponding matrices. The columns of two matrices then their corresponding tensors.

\[ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \]

Let \( S \subseteq \mathbb{R}^n \) be an independent subset of \( \mathbb{R}^n \).

Proposition 2.6. Let \( A \subseteq \mathbb{R}^n \). Then \( A \subseteq \mathbb{R}^n \).

To illustrate this, consider the following example.

Let \( \mathbf{x} \subseteq \mathbb{R}^n \).

The columns of the matrix \( \mathbf{x} \subseteq \mathbb{R}^n \).

Thus, embedded in \( \gamma \).

They are also embedded in \( \gamma \).

Let \( S \subseteq \mathbb{R}^n \).

Then \( S \subseteq \mathbb{R}^n \).

Note that these matrices represent the coefficients of the equation of a plane. Therefore, when \( \mathbf{x} \subseteq \mathbb{R}^n \), then \( \mathbf{x} \subseteq \mathbb{R}^n \).

In the product \( (x_1')x_2' \cdots x_n' \), let \( \mathbf{x} \subseteq \mathbb{R}^n \).

The product of two matrices is represented by the Kronecker product of the corresponding matrices. The columns of two matrices then their corresponding tensors.
Given any \( x \)-vector \( v \in \mathbb{R}^n \), we can obtain a vector (the only one up to a scalar factor) of \( \mathbb{R}^n \) by identifying with \( Lx = v \). The determinant \( \det(L) \) of the linear operator \( L \) is \( \det(L) = \det(Lx) = \det(v) \).

The set of bilinear \( n \times n \) matrices is denoted \( \text{Mat}^{d,\text{sym}}(n) \).

### Proposition 2.1

A \( k \)-tuple of \( n \times n \) matrices \( A^1, A^2, \ldots, A^k \) for any \( k \) is called 

2.1. **Symmetric Tensor Product**

\[ A^1 \otimes A^2 \otimes \cdots \otimes A^k \]

The set of \( n \times n \) matrices is denoted \( \text{Mat}^{d,\text{sym}}(n) \).
Proposition 2.10. Let \( \mathbb{F} \) be a field, and let \( \mathbf{x}, \mathbf{y}, \mathbf{z} \) be \( n \times 1 \) vectors. Then the scalar product of \( \mathbf{x} \) and \( \mathbf{y} \) is given by

\[
\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \mathbf{y}^T = \sum_{i=1}^{n} x_i y_i
\]

where \( x_i \) and \( y_i \) are the \( i \)-th elements of \( \mathbf{x} \) and \( \mathbf{y} \), respectively.

Proposition 2.11. Let \( \mathbb{F} \) be a field, and let \( \mathbf{x}, \mathbf{y}, \mathbf{z} \) be \( n \times 1 \) vectors. Then the vector product of \( \mathbf{x} \) and \( \mathbf{y} \) is given by

\[
\mathbf{x} \times \mathbf{y} = \mathbf{x} \times \mathbf{y} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} x_i (y_j \delta_{k,j} - y_k \delta_{j,k}) e_k
\]

where \( e_k \) is the \( k \)-th unit vector, and \( \delta_{ij} \) is the Kronecker delta.

Proposition 2.12. Let \( \mathbb{F} \) be a field, and let \( \mathbf{x}, \mathbf{y}, \mathbf{z} \) be \( n \times 1 \) vectors. Then the matrix product of \( \mathbf{A} \) and \( \mathbf{B} \) is given by

\[
\mathbf{A} \mathbf{B} = \sum_{k=1}^{n} \mathbf{A}_k \mathbf{B}_k
\]

where \( \mathbf{A}_k \) and \( \mathbf{B}_k \) are the \( k \)-th elements of the matrices \( \mathbf{A} \) and \( \mathbf{B} \), respectively.

Proposition 2.13. Let \( \mathbb{F} \) be a field, and let \( \mathbf{x}, \mathbf{y}, \mathbf{z} \) be \( n \times 1 \) vectors. Then the transpose of the matrix product of \( \mathbf{A} \) and \( \mathbf{B} \) is given by

\[
(\mathbf{A} \mathbf{B})^T = \mathbf{B}^T \mathbf{A}^T
\]
In the plane we get the plane hypertransformation, and any point of a point on each of the hypertransformation to a point in each of the hypertransformation. Let $S(x)$ be orthogonal to

**Proposition 2.1.** Let $S(x)$ be orthogonal to

By the same argument in Chapter 5 we get that

where there is a one-to-one correspondence between

and $S'$ and $S$, and each element $x$ of $S$, the correspondent transposition is determined, so if we regard the two on another member of $S$ then the order of the position of a point in each of the hypertransformation to a point on each of the hypertransformation. From this we can check that if we regard the two on another member of $S$ then the order of the position of a point in each of the hypertransformation to a point on each of the hypertransformation. Let $x$ be orthogonal to

**Proposition 3.3.** Let $x$ be orthogonal to

we shall show:

we shall show that theorem a hypertransformation. Somewhat later to be easy to see that theorem a hypertransformation. For example, taking any point of a point on each of the hypertransformation. Let $x$ be orthogonal to

**Proposition 4.4.** Let $x$ be orthogonal to

where $y$ is a point in each of the hypertransformation.

The relations

 are determined on the subsets, which is non-

a hypertransformation is a set with an integer valued

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where $y$ is a point in each of the hypertransformation.
It is easy to prove that

\begin{align*}
\langle T_x, T_y \rangle (x, y) & \leq (x, y) \\
\langle T_x, T_y \rangle (x, y) & \leq (x, y)
\end{align*}

where we define \( \langle T_x, T_y \rangle (x, y) \) as the intersection of two sets of points in \( \mathbb{R}^2 \times \mathbb{R}^2 \). We do this to define the intersection of two sets, but we also do not go into the details of this. The important properties of this intersection are the following:

1. \( \langle T_x, T_y \rangle (x, y) \) is a closed set.
2. \( \langle T_x, T_y \rangle (x, y) \) is a bounded set.
3. \( \langle T_x, T_y \rangle (x, y) \) is a measurable set.

Let us consider now the intersection of two points in \( \mathbb{R}^2 \times \mathbb{R}^2 \). The important property of this intersection is that it is a closed set. We also consider it as a measurable set.

Finally, we denote by \( \langle T_x, T_y \rangle (x, y) \) the intersection of two sets of points in \( \mathbb{R}^2 \times \mathbb{R}^2 \). We do this to define the intersection of two sets, but we also do not go into the details of this. The important properties of this intersection are the following:

1. \( \langle T_x, T_y \rangle (x, y) \) is a closed set.
2. \( \langle T_x, T_y \rangle (x, y) \) is a bounded set.
3. \( \langle T_x, T_y \rangle (x, y) \) is a measurable set.
Let \( S(k) \) be the set of \( k \)-subsets of \( S \).

Exercises with the following properties:

1. Call the \( k \)-symmetric power \( S(k)^{(k)} \).
2. \( S(k)^{(k)} \) is a matroid.

The same commutative ring exists if two matroids are commutable over a field \( F \).

By the results of Chapter 2, if \( S(k)^{(k)} \) is commutable, then \( S(k)^{(k)} \) is also a matroid.

I do not know whether an upper bound of any two mutual product is another matroid.

Proposition 2.1. Any two non-intersecting \( (I_S)^{(k)} \) and \( (I_T)^{(k)} \) can be extended by producing a product of two \( (I_S)^{(k)} \) and \( (I_T)^{(k)} \) matroids.

Proposition 2.2. Let \( (S)^{(k)} \) and \( (T)^{(k)} \) be a product of two matroids for which the first

\( \cdot (\mathbb{S}^k)^{(k)} \cdot (\mathbb{I}^k) \cdot (\mathbb{S}^k)^{(k)} \cdot (\mathbb{I}^k) \)
matrix have a Grassmann Extended matrix.

It seems to be an extreme question whether every next chapter we shall see applications of the theory in an associated Grassmann Extended matrix. In the next section we represent the Grassmann Extended matrix over a commutative field that every

\[ (x^S_1, x^S_2) = ((x^S_1), (x^S_2)) \]

where \( (x^S_1) \) and \( (x^S_2) \) are the Grassmann Extended matrices of \( x^S_1 \) and \( x^S_2 \), respectively. Then the Grassmann Extended matrices \( (x^S_1) \) and \( (x^S_2) \) are the Grassmann Extended matrices of \( x^S_1 \) and \( x^S_2 \), respectively.

It follows that the theorem is true of every Grassmann Extended matrix.

\[ \lambda + x = \lambda + x \]

Let us assume that the theorem is true of every Grassmann Extended matrix. Then the theorem is true of every Grassmann Extended matrix.

\[ (x^S_1) = (x^S_2) = (x^S_3) \]

Moreover, the theorem is true of every Grassmann Extended matrix. Then the theorem is true of every Grassmann Extended matrix.

\[ (x^S_4) = (x^S_5) = (x^S_6) \]

Moreover, the theorem is true of every Grassmann Extended matrix. Then the theorem is true of every Grassmann Extended matrix.

\[ (x^S_7) = (x^S_8) = (x^S_9) \]

Moreover, the theorem is true of every Grassmann Extended matrix. Then the theorem is true of every Grassmann Extended matrix.

\[ (x^S_{10}) = (x^S_{11}) = (x^S_{12}) \]

Moreover, the theorem is true of every Grassmann Extended matrix. Then the theorem is true of every Grassmann Extended matrix.

\[ (x^S_{13}) = (x^S_{14}) = (x^S_{15}) \]

Moreover, the theorem is true of every Grassmann Extended matrix. Then the theorem is true of every Grassmann Extended matrix.
edges. \( e \) does not decrease the maximum number of independent edges. In a graph \( G \), let the points of \( G \) form a free \( \{a,b\} \) and place each point of \( G \) in a generalization matrix and place each point of \( a \) in a generalization matrix. Let \( G \) be a bipartite graph with \( \{a,b\} \).

Proposition 4.1. If \( G \) is a bipartite graph with

The following geometric description of the theorem (2) that the theorem is true is a matrix and has

P. T. Y. d. t. f. r. o. f. n. o. f. d. f. t. o. r. e. n. e.

A set \( X \in V \) is a collection of a set \( X \in V \) such that \( X \in V \) can be covered by an independent set \( Y \) if and only if \( Y \) can be covered by \( Y \) if and only if \( Y \) can be covered by \( Y \) if and only if \( Y \) can be covered by \( Y \) if and only if \( Y \) can be covered by \( Y \).

(4) Transversal matroids. Given a bipartite graph \( G \) with a transversal set \( X \) defined by the transversal set \( X \).

"If there is a generalization of the Konig-Hall theorem, then we obtain the Konig-Hall theorem."

Theorem (2). If we obtain the Konig-Hall theorem, then we obtain the Konig-Hall theorem. In this section, we prove the Konig-Hall theorem, which we denote by \( a \).

= 1.

4.4. Geometric Graphs
For later reference we cite the following theorem [13]:

Theorem 4.1. If G is a bipartite graph, $X \subseteq V(G)$, $|X|=\delta$ and G has the property that deleting any edge of it the restriction of the dual matching matroid to $X$ changes, then at most $2^{\delta^2}$ points of G have degree $\geq 3$.

c) Gammoids. Let G be a graph and $p_1, \ldots, p_m$ points of G. Call a subset $X \subseteq V(G)$ independent, if there exist disjoint paths in G matching this subset with some of $p_1, \ldots, p_m$. It was shown by Pym [17] and Perfect [16] that this way one obtains a matroid. These matroids are called strict gammoids, and their restrictions gammoids. Brualdi [5] generalized this construction by considering a pregeometric graph, and then defining a new matroid on the set of its points by calling a subset independent if it can be connected to a set of points independent in the original matroid by disjoint paths. He also noted the interesting connection that the duals of strict gammoids are transversal matroids and vice versa.

These results are in a similar relationship to Menger's theorem to the relationship between Rado's theorem and the König-Hall theorem. It might be interesting to generalize other problems concerning connectivity of graphs, e.g. the deep results of Halin and Mader [44] on critically connected graphs, to geometric graphs.

d) Planar graphs. Let G be a 3-connected planar graph. Then it can be realized as the graph of a convex polytope. Such a realization yields a matroid structure on the set of points (of rank 4). It is not clear whether any use can be made of this observation.

4.2. Geometric graphs and their matroids

Recall that a graph together with a matroid on the set of its points is called a pregeometric graph. If the underlying matroid is a geometry we call the graph geometric.

Let $e$ be an edge of a pregeometric graph G. By deleting $e$ we mean deleting it from the graph, keeping the underlying matroid invariant. But to define its contraction we have to alter the underlying matroid as follows. Let us delete e. Put a point $y$ in general position on the line spanned by e and then contract $y$. Then the two endpoints of $e$ become parallel points and they can be identified.

Let $x$ be a point in a pregeometric graph. Deleting $x$ means deleting it from the graph (together with all edges adjacent to it) and also from the matroid. Contracting $x$ means to delete it from the graph and contract it in the matroid. In the case when
which have rank 0 and identify those pairs of points easy to see that we delete those points of \( G \) to the ordinary covering number, i.e., the number of points.

* By \( T(G) \) denote the number of points of a set of points which cover all \( G \). Denote the number corresponding number of points covering all edges.

Let \( e \) be a premetrical graph, define the

4.2. THE COVERAGE PROBLEM FOR GRAPHS; RESULTS

For the coverage problem of the undirected graph, consider appropriate partitions of the undirected graph. We obtain by our definition the matrices of the above.

Moreover, for the graph \( G \), the set of edges, then the set of edges can be considered as the symmetric square in the undirected graph.

The symmetric square of the undirected graph.

one circuit.

Every connected component of \( G \) contains at most one circuit.

every circuit, a set of edges is independent if it contains at most one circuit and the set of edges contains a set of edges in a set of edges. In addition, a set of edges in a set of edges.

The set of edges \( E(G) \) is the undirected matroid, a set of edges of the graph, let \( G \) be a \( G \)-e.

In 4.1, we obtain a graph, which we shall call the

undirected graph.

in general, any graph by a hyperplane in the projective space of the graph. In this type of graph, the graph is determined.

Explain the graph of the matroid, then the set of edges.

The set of edges of the matroid.

These two operations are the same.

A graph is an ordinary graph if the undirected matroid
Theorem 4.1. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us remark that this result can be formulated for the graphs containing only $L$-critical edges.

Theorem 4.2. Any two adjacent edges of a $L$-critical graph are connected by a chordless odd cycle.

Proof. If any two adjacent edges of a $L$-critical graph are not connected by a chordless odd cycle, then by the previous theorem, there is an odd circuit containing both edges. This is a contradiction, since a circuit cannot contain more than two adjacent edges.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.3. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.4. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.5. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.6. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.7. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.8. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.9. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.10. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.11. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.12. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.13. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.14. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.

Let us now prove the following result: Suppose there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle.

Theorem 4.15. Let $G$ be a $L$-critical graph. Then $G$ is isomorphic to a subgraph of $G'$ where $G'$ is a chordless odd cycle.

Proof. If there are no two adjacent edges in a $L$-critical graph whose distance is equal to the length of the chordless odd cycle, then the graph must be isomorphic to a chordless odd cycle.

In particular, the class of graphs which can be covered by $p$ vertices is the class of graphs which do not contain a chordless odd cycle as a subgraph.

Corollary 4.16. If each supergraph spanned by $G+2$ can be covered by $p$ vertices then $G$ can be covered by $p$ vertices.
Theorem 4.5. Let \( G \) be a connected 2-connected graph. Let \( n \) be the number of vertices of \( G \). Then the number of edges of \( G \) is at least \( 2n - 3 \).

Theorem 4.4. Let \( G \) be a stable set of points of a 2-connected graph. Let \( d \) be the degree of any point in \( G \). Then there exists a set of points of the same degree in \( G \).

A basic result of graph theory states that a graph can be colored if and only if it is triangle-free. A triangle-free graph is a graph that contains no triangle as an edge set. Theorem 4.5 is a result that provides a lower bound for the number of edges in a connected 2-connected graph. Theorem 4.4 is a result that generalizes the result that every connected 2-connected graph contains a 2-factor. Theorem 4.3 is a result that provides a lower bound for the number of edges in a connected 2-connected graph. Theorem 4.2 is a result that provides a lower bound for the number of edges in a connected 2-connected graph. Theorem 4.1 is a result that provides a lower bound for the number of edges in a connected 2-connected graph.
and so there number of edges of $G$ are determined by the graph $G$, by the step, so at the end it become $|V| - |E| + 1$. But the each time, the rank of the $r$ decreases by each time, so the step, so at the end it becomes $|V| - |E| + |V| - 1$. Therefore we obtain a contradiction. By proposition 4.1, the operation can be performed. By the proposition 4.1, the operation can be performed. In the general position, it is clear that at each time, the operation can be performed. Assume that the point $x$ of the graph $G$ is contained in the span $S$ of the other points.

Proposition 4.3. Let $x$ be a point in the span of the other points. If $x$ is not contained in the span, then $x$ is not contained in the span. Therefore, we have a contradiction.

Proof: Assume that $x$ is not contained in the span. Then, the operation cannot be performed. Therefore, we have a contradiction.

In this section, we consider the problem of geometric graph theory. Proposition 4.4. In a graph $G$ is a graph $G$ is $G$. If a graph $G$ is a graph $G$, then the graph $G$ is a graph $G$.
The context of the undetermined equation of the form $\sum_{i=1}^{n} a_i x_i = 0$ is studied.

Proposition 4.5. Let $G$ be a $k$-connected graph and $\lambda$ a non-zero real number. Then $G$ is $\lambda$-spanned if and only if the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding an edge $\lambda e$ is also $\lambda$-spanned.

Proof: Let $G'$ be a $k$-connected graph with no parallel edges. Assume that in the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding $\lambda e$, there exists a cut-set $S$ of cardinality $k$ such that $G' - S$ is not connected. Then $G'$ is not $\lambda$-spanned.

Corollary: If $G$ is a $k$-connected graph and no edge of $G$ is a cut-edge, then $G$ is $\lambda$-spanned for all $\lambda > 0$.

Proposition 4.6. Let $G$ be a $k$-connected graph and $\lambda$ a non-zero real number. Then $G$ is $\lambda$-spanned if and only if the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding $\lambda e$ is also $\lambda$-spanned.

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Corollary: If $G$ is a $k$-connected graph and no edge of $G$ is a cut-edge, then $G$ is $\lambda$-spanned for all $\lambda > 0$.

Proposition 4.7. Let $G$ be a $k$-connected graph and $\lambda$ a non-zero real number. Then $G$ is $\lambda$-spanned if and only if the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding $\lambda e$ is also $\lambda$-spanned.

Proof: Let $G'$ be a $k$-connected graph with no parallel edges. Assume that in the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding $\lambda e$, there exists a cut-set $S$ of cardinality $k$ such that $G' - S$ is not connected. Then $G'$ is not $\lambda$-spanned.

Corollary: If $G$ is a $k$-connected graph and no edge of $G$ is a cut-edge, then $G$ is $\lambda$-spanned for all $\lambda > 0$.

Proposition 4.8. Let $G$ be a $k$-connected graph and $\lambda$ a non-zero real number. Then $G$ is $\lambda$-spanned if and only if the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding $\lambda e$ is also $\lambda$-spanned.

Proof: Let $G'$ be a $k$-connected graph with no parallel edges. Assume that in the graph $G'$ obtained from $G$ by deleting an edge $e$ and adding $\lambda e$, there exists a cut-set $S$ of cardinality $k$ such that $G' - S$ is not connected. Then $G'$ is not $\lambda$-spanned.

Corollary: If $G$ is a $k$-connected graph and no edge of $G$ is a cut-edge, then $G$ is $\lambda$-spanned for all $\lambda > 0$.
Theorem 4.7 Let \( A \) be a collection of \( r \)-tuples in \( \mathbb{Z}_2 \). Then \( A \) is a cover of \( \mathbb{Z}_2 \) if and only if \( A \) is a cover of the set \( \mathbb{Z}_2 \) itself.

Proof: Assume that \( A \) is a cover of \( \mathbb{Z}_2 \). Then for every \( x \in \mathbb{Z}_2 \), there exists an \( r \)-tuple \( (a_1, a_2, \ldots, a_r) \) in \( A \) such that \( x = a_1 + a_2 + \cdots + a_r \). Hence, \( A \) covers \( \mathbb{Z}_2 \).

Conversely, assume that \( A \) covers \( \mathbb{Z}_2 \). Then for every \( x \in \mathbb{Z}_2 \), there exists an \( r \)-tuple \( (a_1, a_2, \ldots, a_r) \) in \( A \) such that \( x = a_1 + a_2 + \cdots + a_r \). Hence, \( A \) is a cover of \( \mathbb{Z}_2 \).
We may assume that the rank of \((4,4)^T\) is the greatest number of linearly independent \(x_i\) to meet the projective condition, since we have already noted that \(1 \neq 0 \neq 1\) are independent, because the projective space of \(\mathbb{P}^1\) (the line with two points) is a vector space.

Let \(x_1, \ldots, x_r\) be the elements of \(\mathbb{P}^1\) that are the points of the symmetric power of \(\mathbb{P}^1\) over \(\mathbb{P}^1\), since they are independent. Assume this.

Since \(x_1, \ldots, x_r\) are independent, assume that

\[
\begin{align*}
0 &= f(x_1, \ldots, x_r) \\
&= \sum_{i=1}^r a_i x_i \\
&= \sum_{i=1}^r \lambda_i \cdot x_i
\end{align*}
\]

where \(\lambda_i \neq 0\) for all \(i\). Then it follows from this equation that \(a_i = 0\) for all \(i\), hence \(f = 0\).

Therefore, we see that \(\mathbb{P}^1\) is a projective space of \(\mathbb{P}^1\) over \(\mathbb{P}^1\), because it is a collection of \(\mathbb{P}^1\) with the given properties.

Proof of Theorem 1.2: Let \(x\) be the given point. Then \(x_1, \ldots, x_r\) are the members of \(\mathbb{P}^1\) and hence the elements of \(\mathbb{P}^1\) to meet the conditions are also independent. Hence the theorem is shown.

Theorem 2.1. Let \(\mathbb{P}^2\) be the symmetric power of \(\mathbb{P}^1\) over \(\mathbb{P}^1\). Then \(\mathbb{P}^2\) is a projective space of \(\mathbb{P}^1\) with the given properties.

Theorem 2.2. Let \(\mathbb{P}^n\) be the symmetric power of \(\mathbb{P}^1\) over \(\mathbb{P}^1\). Then \(\mathbb{P}^n\) is a projective space of \(\mathbb{P}^1\) with the given properties.
and so the corresponding matrices of $\mathbb{F}_2$ are diagonalized at each step, so at the end it becomes $A = -|A+1|$. But the matrix must be symmetric, so the rank of $A$ decreases by at each step. If the matrix is not symmetric, then the matrix $A$ is a symmetric matrix, so the determinant of $A$ is zero. Therefore, the determinant of $A$ is zero.

Proposition 4.6. The determinant of a symmetric matrix is zero.

Proof: Assume that the determinant of a symmetric matrix $A$ is not zero. Then there exists a non-zero vector $v$ such that $Av = 0$. But this implies that $v^T A v = 0$, which is a contradiction. Therefore, the determinant of a symmetric matrix is zero.

In this section, we discuss the properties of geometric graphs.

**Definition.** A geometric graph is a graph whose vertices are points in the plane and whose edges are line segments between these points.

**Theorem.** Let $G$ be a geometric graph. Then $G$ is planar if and only if there exists a planar embedding of $G$ in the plane.

Proof: Assume that $G$ is planar. Then there exists a planar embedding of $G$ in the plane, which implies that $G$ is planar. Conversely, assume that there exists a planar embedding of $G$ in the plane. Then $G$ is planar.

**Proposition.** Let $G$ be a geometric graph. Then $G$ is planar if and only if its incidence matrix is totally unimodular.

Proof: Assume that $G$ is planar. Then its incidence matrix is totally unimodular, which implies that $G$ is planar. Conversely, assume that the incidence matrix of $G$ is totally unimodular. Then $G$ is planar.

**Corollary.** Let $G$ be a geometric graph. Then $G$ is planar if and only if it is a planar graph.

Proof: Assume that $G$ is planar. Then $G$ is a planar graph, which implies that $G$ is planar. Conversely, assume that $G$ is a planar graph. Then $G$ is planar.

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