Open problems in polyhedral combinatorics

Alexander Schrijver

CWI and University of Amsterdam

E. Egerváry 1931: min-max formula for maximum weight perfect matching in a complete bipartite graph

- first result in *polyhedral* combinatorics

T.E. Easterfield 1946: $O(2^n n^2)$ algorithm for optimum assignment

J. Robinson 1949: cycle cancelling method for optimum assignment

H.W. Kuhn 1955: Hungarian method

- first polynomial-time method
- primal-dual
- iterative shortest path method

Open problems in polyhedral combinatorics

NP$\neq$P?

Can a maximum flow be found in $O(nm)$ time?

Is the matching polytope the projection of a polytope with a polynomial number of facets?

Woodall's conjecture (1978): The minimum size of a directed cut in a directed graph is equal to the maximum number of disjoint directed cut covers.

Give a combinatorial polynomial-time algorithm finding a maximum stable set in a perfect graph.

Is there a polyhedral extension of Mader's T-paths theorem?

Let G be an undirected graph and let $T \subseteq VG$.

A T-*path* is a path connecting two distinct vertices in T.

Mader 1978:

maximum # of internally disjoint T-paths = minimum of ...

- polyhedral?
- algorithmic?
- matroidal?