Contagion and interdependence on the CDS market: a Bayesian Factor Model approach

Author: Dániel Biró
Supervisor: Milán Csaba Badics

May 9, 2019
Acknowledgement

Here I would like to thank the guidance of my supervisor Milán Csaba Badics, who enthusiastically helped me with his professional and academic knowledge. I would also like to thank my family, who always supported me during the years I spent at the University and gave me a calm and safe environment to pursue my studies.
Contents

1 Introduction 4

2 Literature review 6

3 Research frameworks 9

4 Bayesian factor model with stochastic volatility 11
   4.1 A general dynamic factor model with stochastic volatility . . . . . . . 11
   4.2 Bayesian econometrics . . . . . . . . . . . . . . . . . . . . . . . . . . . 12
   4.3 Model specification . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
      4.3.1 Contagion and interdependence . . . . . . . . . . . . . . . . . 16
      4.3.2 Normalization and identification . . . . . . . . . . . . . . . . . 17
      4.3.3 Priors . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
      4.3.4 Estimation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 20

5 The Data 25

6 Empirical results 28
   6.1 Estimated factors . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
   6.2 Evidence of contagion and interdependence . . . . . . . . . . . . . . . 29
   6.3 Sources of contagion . . . . . . . . . . . . . . . . . . . . . . . . . . . 36

7 Robustness tests 41

8 Future research 43

9 Conclusion 43
List of Tables

1 List of the parameters to be estimated with their priors and initial values ........................................ 20
2 Country level descriptive statistics ........................................ 49

List of Figures

1 Credit default swap spread of 8 analyzed countries (Greece and Venezuela on secondary axis) ................. 26
2 Component loadings obtained from the principal component analysis ........................................ 27
3 Posterior means of the dynamic factors with 95% highest probability intervals ........................................ 29
4 Posterior means of the cumulated dynamic factors with 95% highest probability interval of the spread changes and benchmark factors .... 30
5 Posterior means of $V_{i,t}^{G,c}$ and $V_{i,t}^{C}$ with 90% highest probability intervals ............................... 31
6 Posterior means of European $V_{i,t}^{R,c}$ and $V_{i,t}^{R}$ measures with 90% highest probability intervals ............ 33
7 Posterior means of Latin American and Southeast Asian $V_{i,t}^{R,c}$ and $V_{i,t}^{R}$ measures with 90% highest probability intervals .......... 33
8 Stacked area plot of the posterior means of $V_{i,t}^{G,c}$ and $V_{i,t}^{R,c}$ ..................................................... 35
9 Mean of the corrected variances explained by the global and regional factors ........................................ 36
10 Posterior means of the permanent and aggregated stochastic volatility of the factors with 90% highest probability intervals ....................... 37
11 Posterior means of the permanent and aggregated idiosyncratic stochastic volatility with 90% highest probability intervals ......................... 38
12 Posterior means of the global loadings with 90% highest probability intervals ........................................ 39
13 Posterior means of the regional loadings for European countries with 90% highest probability intervals .......... 40
14 Posterior means of the regional loadings for Latin American and Southeast Asian countries with 90% highest probability intervals .... 41
15 Average market integration measure differences from the original results ........................................ 42
1 Introduction

The Credit Default Swap (CDS) contracts have become widespread in the early 2000s but their role was mostly highlighted in the last global financial crisis and the subsequent European sovereign debt crisis. Credit Default Swaps are functioning as quasi-insurances on financial markets in case of debt defaults. The buyer of protection pays a spread quoted in basis points up to a given maturity with a predetermined frequency (e.g. for 5 years, quarterly), while the seller is willing to take the risk to compensate the buyer if a certain credit event happens. This means, if the contractually fixed credit event (e.g. default of the debtor) happens before maturity, the seller has an obligation to buy the debt on par value, therefore, to bear the credit losses (Augustin et al., 2014).

The CDS instruments are existing in many forms including corporate and bank CDSs with maturities from 3 months up to more than 10 years, but one of the most important of them is the sovereign CDS contract written on the debts of sovereign states. As these CDSs are related to debt securities issued by a given country, their spreads can be used as an indicator of all default related risks of the corresponding country. The last two decade was an eventful period in terms of sovereign risk variations, that attracted the attention of market participants and researchers. In the early 2000s, the CDS market was mainly focused on emerging countries, but the events of the global financial crisis and the European sovereign debt crisis increased the role of the Credit Default Swaps in developed economies.

As the sovereign risk of a country changes in time, its credit spread also varies and quickly reacts to market movements and several global, regional and local factors. The effect of the global factors is often manifested in strong co-movements in CDS spreads. Longstaff et al. (2011) showed the first component from principal component analysis can explain large variation in CDS spreads, and the estimated factor is highly positively correlated with the VIX index and negatively with US stock market returns. Several studies have similar findings on the commonality describing the credit spreads of different countries on various time intervals while highlighting the importance of global and regional economic determinants (Pan and Singleton (2008), Dieckmann and Plank (2011), Hibbert and Pavlova (2017)).

After the Lehman Brothers had collapsed and the Greek debt crisis had revealed, the literature became more focused on the contagion and interdependence phenomena. Although there is no consensus regarding the definition of these concepts, the interdependence is often described as a long-range dependence between a group
of countries in a financial market, while the literature often refers to contagion as change in the connectedness and risk propagation among a set of countries after a shock occurs in one or a subset of countries. The contagion and interdependence effects are well investigated on various traditional financial markets: on stock markets (Forbes and Rigobon (2002), Bekaert and Harvey (2003), Bekaert et al. (2009)), on bond markets (Bunda et al. (2009), Dungey et al. (2006)) between stock and foreign exchange markets within emerging economies (Tai, 2007). Studying contagion and interdependence is also relevant in the sovereign credit markets as they are closely related to the sovereign bond markets, but they explicitly represent the default risk related part of the sovereign yields. Therefore, findings can have implications on monetary policies regarding measures that could mitigate the vulnerability of a sovereign when other countries are experiencing crisis periods. The articles related to sovereign credit risk contagion are often concentrated on credit spreads of European sovereigns at the time of the debt crisis, but the findings are not conclusive.

Earlier studies on the credit risk market contagion applied different methodologies such as corrected correlation measures, VAR models, multi-variate DCC-GARCH specifications and factor models, and some studies find clear indications of contagion in the post-Lehman period and after the Greek debt crisis (Kalbaska and Gatkowski (2012), Broto and Perez-Quiros (2015)), but others show the lack of evidence (Beirne and Fratzsch, 2013). The aim of this thesis is to contribute to the existing literature by applying an advanced methodology, namely estimating a Bayesian Dynamic Factor model with global and regional factors, time-varying parameters and two-component stochastic volatility on a large dataset containing credit spreads from 2005 to 2018 for 40 countries to seek the answer for the following questions: Are there significant global and regional components? If so, how these latent factors evolved during the sample interval? What proportion of the CDS spread variation is explained by the common global and regional factors for each country? Is any group of sovereigns experienced contagion? If so, what makes them different from the rest of the countries and what is the source of contagion?

To our best knowledge, this is the first time when a Bayesian dynamic factor model with time-varying parameters is utilized to study contagion and interdependence on the sovereign credit risk market. The model is specified with a global factor, which influences all country’s CDS spread and 3 regional factors (European, Latin American, and Southeast Asian) to incorporate the rarely addressed differences of credit spreads between geographical regions, while the time-varying setup allows us to identify relevant changes in the connectedness of CDS spread movements.
2 Literature review

After the observed turmoil periods, a significant part of the sovereign risk literature focused on the co-movement of sovereign credit spreads and their economic determinants. Longstaff et al. (2011) found by analyzing the CDS spread of 26 countries between 2000 and 2010 that 64% of the variation can be explained by the first component from principal component analysis on the whole sample, while the explained variation increased to 74% in the last turmoil period. Their estimated first component is strongly related to US stock returns and the VIX index. Pan and Singleton (2008) also showed this connection with US financial indicators in the case of Mexico, Turkey, and Korea. As in the before mentioned articles, the co-movement and the level of CDS spreads are showed to be more explained by global and regional economic variables and less by local factors. Addressing the European CDS spreads, Dieckmann and Plank (2011) found the EuroStoxx 50 index and the German Bund rate, while Fontana and Scheicher (2016) identify the iTraxx Europe index as important economic determinants. Blommestein et al. (2016) respond to these results by using a regime dependent model, showing the importance of different determinants is time-dependent. Furthermore, the economic determinants for the Eurozone credit spread market identified by Bratis et al. (2018) are the VSTOXX index, the European variance risk premium and the elections in Greece and Spain. One might think that more economic connectedness explains more co-movements in the financial markets but Ang and Longstaff (2013) found that less systemic risk can be estimated from U.S. sovereigns than European ones and as other studies showed, the commonality of the European sovereign risk is explained by financial variables.

Changes in the connectedness and linkages of the financial markets are often described by the so-called contagion effect, although there is no consensus in the definition of contagion in the existing literature. It is often defined as “a significant increase in cross-market linkages after a shock to one country (or group of countries)” (Forbes and Rigobon, 2002), but many similar definitions are applied. Dungey and
Martin (2001) describes contagion as a “spillover effect of unanticipated con-tempo-
aneous movements across countries”. Kalbaska and Gatkowski (2012) and Caporin
et al. (2018) have similar definitions stated as the increased shock propagation and
excess correlation in crisis compared to normal periods. Others, as Bekaert and
Harvey (2003) and Beirne and Fratzscher (2013) connect the contagion to economic
fundamentals and describe it as an unexpected excess correlation and a significant
change of how these fundamentals are priced. In our study, we utilize a similar
definition as Broto and Perez-Quiros (2015) who defines contagion as “a significant
variation in the cross-country co-movement of CDS spreads, compared with that
of non-crisis periods, triggered by a specific country or group of countries”. For a
comprehensive review of definitions see Missio and Watzka (2011) and Bratis et al.
(2018).

In the early studies, the concept of interdependence is often used as a synonym
to contagion but Forbes and Rigobon (2002) clearly isolated the two phenomena.
As observed, certain financial markets of various countries tend to move together
similarly in normal periods and in major upheavals. Based on the above-described
definitions, this situation cannot be considered contagion, because there was no
significant change in the linkage and connectedness of these markets. Therefore, when
the connectedness among two or more entities is substantially high regardless of the
economic conditions, the phenomenon is called interdependence.

Since the volume of the CDS market has been started growing remarkably, two
events seemed to be an obvious case of possible contagion: the global financial
crisis and the European sovereign crisis, while the interdependence can be tested
on the whole time period. As an evidence, Wang and Moore (2012) document the
significant effect of the Lehman collapse on the CDS markets. In their article, the
credit spread movements of the 38 examined sovereigns have become more integrated
after September 2008, which shift was mainly led by the U.S. interest rates. The study
of Kalbaska and Gatkowski (2012) takes the beginning of the market integration even
earlier to the credit crunch related to BNP Paribas in August 2007. They showed
a high average increase in the exponentially weighted pairwise correlation among
European credit spreads after August 2007, but the correlations also peaked after
the Lehman crisis.

As a reaction to the U.S. financial crisis, several European governments offered
rescue packages to their national banking system, which decreased the credit risk of the
banking sector, but the risk transferred to the rescuing sovereign’s countries (Ejsing
and Lemke, 2011). After October 2009 when it has been announced that Greece
intentionally reported false debt statistics, the Greek CDS spread quickly increased along with the credit spread of other Eurozone periphery countries that had high debts and similar fiscal scheme as Greece, like Spain and Portugal, whereas Ireland was affected by the burst of its real estate bubble. Kalbaska and Gatkowski (2012) show that these events are mirrored in credit spread contagion waves throughout the Eurozone from 2009 to 2010. They found the increase of the connectedness among the PIIGS (Portugal, Italy, Ireland, Greece and Spain) and core countries measured by Granger causality as a sign of contagion, while it is also shown that the PIIGS countries can also trigger the sovereign spread of the core countries, but they are - especially Portugal - more vulnerable to outside shocks.

Although the presented studies find evidence of contagion, Beirne and Fratzscher (2013) claim that the global rise of the sovereign credit spreads during the European debt crisis are not due to contagion effect but explained by worsening of sovereign financial fundamentals and the increased sensitivity to them. Instead of credit spreads, Caporin et al. (2018) consider sovereign bond spreads and find no contagion between 2003 and 2013 in the Eurozone countries. Furthermore, the synchronization among Eurozone sovereigns decreases in the post-Lehman period - based on their reasoning - caused by changes in market perceptions after the US financial crisis. On the other hand, Argyrou and Kontonikas (2012) find strong evidence of contagion also inferred from sovereign bond spreads.

Sabkha et al. (2018) provide more finding to confirm the existence of contagion by applying bivariate AR-FIEGARCH-DCC models to test how the correlation between credit spread movements are changed. They found increased co-movement with the sovereign credit spreads of the US, Greece, and Ireland at most of the investigated 35 countries, both after the global financial crisis and the European debt turmoil regardless of their economic situation (developed, newly industrialized, PIIGS or emerging) and geographic location. The separation of contagion and interdependence in the CDS markets of the Eurozone was addressed by Bratis et al. (2018), who found weak interdependence among the 9 analyzed sovereigns pre- and post-crisis, but contagion is also identified during the sovereign crisis, while Ireland is shown to be affected by shocks from the U.S. market. They also confirmed the feedback loop of the risk transformation between the sovereigns and the credit risk of their banks, which is in line with previous findings (Alter and Schüler (2012), Dieckmann and Plank (2011)).

Broto and Perez-Quiros (2015) claim, that the credit spread market among core and periphery countries can be described with only one latent factor during normal
periods, but after the Greek CDS spread overcome the credit spreads of many other sovereigns, two factors are required. The first factor explains the usual co-movement between international credit risk markets, while the second is the main driver of contagion after the fall of 2009, which factor was fed by not only Greece but Portugal and Spain too, based on their dynamic factor model specification.

The differences between large geographical regions are less investigated. Longstaff et al. (2011) identify the second principal component as the difference between European and Latin American credit spreads but a more detailed analysis is carried out by Hibbert and Pavlova (2017). They found that Latin American credit spreads show more commonality and explained to a greater extent by global variables such as long-term bond rates and market volatility, while the European CDS spreads also depend on TED spread changes and credit rating changes in the upper quantiles. Sabkha et al. (2018) is also a notable exception, who find the shocks after the Greek crisis spreads globally, regardless of the economic development of the related countries.

3 Research frameworks

An evident consequence of the disagreement about the exact definition of contagion and interdependence is that researchers have used a wild range of methods to detect and measure the existence and magnitude of the connectedness in financial markets. The principal component analysis could be a starting point of an analysis to describe the co-movement among financial time series as in Longstaff et al. (2011), but is not capable to disentangle between contagion and interdependence effects. In their groundbreaking article, Forbes and Rigobon (2002) propose a corrected correlation to measure how the unconditional correlation evolves in the stock markets after the crisis in the U.S., Mexico and Asia in the years of 1987, 1994 and 1997 respectively. To evaluate whether the unconditional correlations have changed, their test corrects for the heteroscedasticity bias known as a spurious increase in the correlations when financial markets experience upheaval periods, even though the underlying dynamics between participants stayed the same. Using the introduced test, Andenmatten and Brill (2011) find interdependence within the CDS market, while the increase in the corrected correlation indicates the presence of contagion after the Greek debt crisis.

As the definition of contagion can be related to the change in the pricing of fundamentals, some authors investigate how the sensitivity to these fundamentals changed and whether excess correlations, not explained by the underlying funda-
mentals occurred (Beirne and Fratzscher (2013), Bekaert and Harvey (2003)). Other studies applied direct time series models to capture the contagion effect or to find overwhelming evidence of interdependence on the sovereign and bank CDS market. VAR and GARCH type of models are popular choices (Kalbaska and Gatkowski (2012), Wang and Moore (2012), Bratis et al. (2018), Sabkha et al. (2018)), because they offer a straightforward way to model and measure the connectedness between financial time series. However, explicitly modeling the correlation structure of a large number of time-series is difficult, hence most of the above-mentioned methods rely on pairwise correlation comparisons.

The utilization of factor models can circumvent the need for high dimensional correlation structures. Furthermore, Dungey and Martin (2001, 2007) argued that identifying proxy economic variables describing the common factors of financial time series is difficult as one needs to select a set of indicators that might have a different role in normal times than crisis periods. Specifying a model for credit spreads as a latent factor model eliminates the necessity of choosing economic variables determining the co-movement of CDS spreads. The difficulties related to the large correlation matrixes and the not certainly known economic determinants are eliminated, but one might still have to pre-select a group of shock propagating countries to explicitly incorporate their effect in the specified models. Broto and Perez-Quiros (2015) provide an alternative solution to select which countries feed the contagion effect. If an estimated factor is clearly related to the increased connectedness among a group of countries, the product of the forecasting error and the Kalman gain can be used to measure the contribution of each country to the contagion factor, although they also emphasize that the method can only be used if a factor arises clearly as contagion factor.

In our analysis, we estimate a model which strongly builds on the existing methods and empirical observations but also employs improved techniques. In order to avoid the aforementioned issues regarding the correlation and economic determinants, we utilize the family of latent factor models as a starting point, while we also let the unobserved factors to change according to an autoregressive process. A similar model was estimated by Diebold et al. (2008) to estimate the commonality in yield curves. However, to capture the changes in the connectedness between sovereign CDS spreads, we need to include time dependency in the estimated parameters as in Del Negro and Otrok (2008). We cover an ever-evolving 14 years period of the sovereign credit risk market, therefore it would be too restrictive to use a model with constant factor loadings, thus we let the loadings to be adjusted in time as the market evolves. Next, we specify the volatility of the error terms in the observation
and factor equation as a two-component stochastic process to incorporate the effect of high volatility periods during a global, regional and idiosyncratic crisis, and to compute a heteroscedasticity bias corrected connectedness measure proposed by Everaert and Pozzi (2016). Finally, a new element of our model is the inclusion of regional factors to analyze the contagion and interdependence effect on both global and regional level. All these enhancements lead us to Bayesian econometric methods, which are commonly used to estimate dynamic factor models with a large number of parameters (Koop et al., 2010). Our model cannot detect the sources of contagion, on this we rely on the existing literature, but the time-varying structure allows us to evaluate the dynamics of interdependence, while the observed structural changes can identify a group of countries that are affected by the contagion phenomena.

4 Bayesian factor model with stochastic volatility

4.1. A general dynamic factor model with stochastic volatility

The application of dynamic factor models is widespread among econometricians (Stock and Watson (1989), Kose et al. (2003), Stock and Watson (2005), Del Negro and Otrok (2008), Del Negro and Schorfheide (2011), Mumtaz and Surico (2012)). As the number of observed time series grows, the estimation of model specifying exact relationships between all variables becomes cumbersome. A convenient solution is offered by factor models by taking advantage on the observation, that N number of time series can be accurately represented by K number of factors where K is much smaller than N. These factors can also be considered latent variables and estimated by econometric techniques. Following the argument of Dungey and Martin (2001, 2007), that factor models can circumvent the need for large correlation matrixes and predetermined fundamentals, we estimate a dynamic factor model described generally here and then specified to our analysis in the next paragraphs. Denote \( y_{i,t} \) the variable of interest for the \( i \)th individual at time \( t \). The dynamic factor models assume \( y_{i,t} \) depends on a constant term, a fraction explained by unobservable latent factors, and an idiosyncratic part:

\[
y_{i,t} = a_i + b_{i,t} \mathbf{F}_t + \epsilon_{i,t},
\]

(1)

where \( a_i \) is the constant term, \( b_{i,t} = (b_{i,1,t}, \ldots, b_{i,K,t}) \) is the factor loading vector at time \( t \), \( \mathbf{F}_t = (f_{1,t}, \ldots, f_{K,t})' \) is the vector of \( K \) latent factors, while the last part represents the idiosyncratic component not explained by the common factors \((i = 1, \ldots, N, \text{and } t = 0, \ldots, T)\). The value of the unobserved factor at time \( t \) is some function of its own lagged values and an error term, usually an autoregressive or
random walk process. One can also assume that factor loadings are not constant in time, therefore they can also be modeled with time dependency as:

\[ f_{j,t} = G(f_{i,j,t-1}, f_{i,j,t-2}, ...) + u_{j,t}, \]  
\[ b_{i,j,t} = F(b_{i,j,t-1}, b_{i,j,t-2}, ...) + v_{i,j,t}, \]  

(2)

(3)

The residual terms in equation (1), (2) and (3) are usually considered as normally distributed errors with 0 mean, but their variance and covariance can be modeled allowing the factor models to have stochastic volatility in each of their equation.

The frequentist time series econometrics also offers estimation methods for dynamic factor models via Gaussian maximum likelihood, Kalman filter, and Expectation-Maximization techniques, but we instead turn to the Bayesian approach to estimate our above-specified model for several reasons. Even with constant volatility in the idiosyncratic and factor components, the maximum likelihood-based methods often perform poorly because of the large number of parameters to be estimated, while the results are heavily depending on the values set as initials at the beginning of the optimization process. However, allowing the volatility of the error terms to stochastically evolve makes the likelihood function intractable and explicitly calls for other techniques, while Bayesian methods are convenient ways of dealing with the time-varying parameters in equation (1).

4.2. Bayesian econometrics

Econometricians and the econometric methods are often divided into two separate groups and approaches called frequentist and Bayesian. The frequentist way handles the unknown parameters as fixed values and estimates are inferred from the observable sample. On the other hand, the Bayesian way looks at the parameters not as fixed values, but as realizations of an underlying distribution estimated by appropriately combining prior beliefs and information extracted from the observed sample. Its main assumption follows the Bayes theorem, stating that the posterior joint density of parameters \( \Theta \) conditioned on the observable sample \( y \) is proportional to the likelihood of sample realization conditioned on parameters times the prior density of the parameters, formally:

\[ p(\Theta|y) \propto p(y|\Theta)p(\Theta), \]  

(4)

analogously, Bayesians seek to answer the question: "What can we learn about the parameters if a sample is given?". This formula expresses how the inferred posterior distribution is related to prior beliefs about the parameters, while it is also updated with information extracted from the observed sample.
As a first step, the practitioner must choose a suitable prior distribution for the parameters and specify the model and calculate the likelihood conditioned on the parameters. Conjugate priors are generally applied to preserve the form of the prior distribution in the posterior distribution to ease the computation and allows to clearly compare posterior and prior distributions, while likelihoods are often based on normal linear models. The main criticism of the Bayesian approach is often related to the choice of arbitrary priors, but as we will discuss later, the dependence on arbitrarily determined priors can be mitigated by using less informative priors to let the data speak for itself, while still preserving other methodological advantages.

Let us first consider the case with only one parameter $\theta$. If one knows $p(\theta | y)$, the expected value of $g(\theta)$ conditioned on the sample $y$ can be calculated using the basics from probability theory:

$$E(g(\theta) | y) = \int g(\theta) p(\theta | y) d\theta$$

(5)

where $g(\theta)$ is the function of interest. The $g(\theta) = \theta$ case is often referred as the posterior mean, while the $g(\theta) = \theta^2$ case allows us to compute the posterior variance. If the posterior density appears in the form of a well-known distribution, these cases are reduced to the theoretical mean and variance of the posterior distribution. Monte Carlo integration methods are also available to make inferences, as one can approximate $E(g(\theta) | y)$ by drawing random $N$ samples from $p(\theta | y)$ and calculate:

$$\hat{g}_N = \frac{1}{N} \sum_{k=1}^{N} g(\theta^{(k)})$$

(6)

where $\theta^{(k)}$ are the random samples and $\hat{g}_N$ converges to $E(g(\theta) | y)$ as $N$ goes to infinity. If drawing random samples from $p(\theta | y)$ is complicated, the Metropolis-Hastings algorithm usually provides a solution. However, if the number of parameters to be estimated is more than one, all the above-mentioned calculations turn into multivariate. When econometricians work with complex models, the dimension of the joint posterior distribution of the parameters becomes large and direct methods are rarely available. Fortunately, it is often easy to calculate and straightforward to draw from the marginal densities $p(\theta_i | y, \theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_M)$. This observation leads to the Gibbs sampler, which is a convenient way to deal with high-dimensional posterior distributions if the marginals are known and easy to draw from them. The Gibbs sampling algorithm has the following steps:

- **Step 0:** set an initial value for all $\theta_i^{(0)}$, $i = 1, ..., M$

Repeat $S$ times ($s = 1, ..., S$):
Step 1: draw a sample $\theta_1^{(s)}$ from $p\left(\theta_1|y, \theta_2^{(s-1)}, \theta_3^{(s-1)}, \ldots, \theta_M^{(s-1)}\right)$

Step 2: draw a sample $\theta_2^{(s)}$ from $p\left(\theta_2|y, \theta_1^{(s)}, \theta_3^{(s-1)}, \ldots, \theta_M^{(s-1)}\right)$

...  

Step M: draw a sample $\theta_M^{(s)}$ from $p\left(\theta_M|y, \theta_1^{(s)}, \theta_2^{(s)}, \ldots, \theta_{M-1}^{(s)}\right)$

Now the drawn values of $\theta_i$ are not independent and depend on the choice of $\theta_i^{(0)}$. However, if $S$ is large enough, under weak conditions the effect of $\theta_i^{(0)}$ is neglected and the samples $(\theta_1^{(s)}, \theta_2^{(s)}, \ldots, \theta_M^{(s)})$ are converging to the joint posterior distribution, while the marginals approximate the posterior marginal distributions and the same Monte Carlo integration method is applicable as described above. Although we described the Gibbs sampler with one sampling step for each parameter, it is often the case that we know the marginals jointly for a set of parameters (e.g. $p(\theta_1, \theta_2|y, \theta_3, \ldots, \theta_M)$). The Gibbs sampling method is still applicable, but now it has a sampling step for each block of parameters, conditioned on all other parameter blocks. It is also a common choice to drop the first $S_0$ samples to make sure we only use those draws for the final inference, for which the process is already converged to the joint posterior distribution (Koop, 2003).

4.3. Model specification

In this section, we specify our Bayesian dynamic factor model regarding the parameter and factor dynamic assumptions, priors, and models identification issues. Our Bayesian approach enables us to model either the level of the variable of interest even with unit-root like properties (Diebold et al., 2008) or the returns/changes in the observed values (Del Negro and Otrok, 2008). Denote $Y_{i,t}$ the observable credit spreads of sovereign $i$ at time $t$, where $i = 1, \ldots, M$, and $t = 0, \ldots, T$. One might be more interested in the level of the CDS spreads, but during our sample, some countries experienced exponential growth in their CDS spreads, which makes it hard to model the level accurately. Following this reasoning, we model the changes in the individual CDS spreads $y_{i,t} = Y_{i,t} - Y_{i,t-1}$, as a sum of a constant term, a fraction explained by unobservable factors, and an idiosyncratic part with stochastic volatility. We set the number of factors equal to 4, as one factor represents the global component, while the other 3 are the regional factors. We allow the global factor to have an effect on each country’s spread changes, but the regional factors are connected only to those CDS spread changes, for which the sovereign is located in the given region. This means equation (1) takes the following form:

$$y_{i,t} = a_i + b_{i,G,t} f_{G,t} + b_{i,R,t} f_{R,t} + \sigma_i^2 e^{2h_i,t} u_{i,t},$$

(7)
where \( f_{G,t} \) and \( f_{R,t} \) the corresponding global and regional factors, whereas \( b_{i,G,t} \) and \( b_{i,R,t} \) are the factor loadings and the last part represents the idiosyncratic component not explained by the common factors, whereas \( \sigma_i^2 \) is the non-time-varying variance and \( \varepsilon_i,h \) is the stochastic volatility and \( u_i,t \) are i.i.d. \( N(0,1) \) distributed errors. Following Everaert and Pozzi (2016), we model the individual stochastic volatilities as a two factor process \( \bar{h}_{i,t} = h_{i,t} + \hat{h}_{i,t} \), where

\[
\begin{align*}
    h_{i,t} &= \rho_i h_{i,t-1} + w_{i,t}, \\
    \hat{h}_{i,t} &= \hat{\rho}_i \hat{h}_{i,t-1} + \hat{w}_{i,t},
\end{align*}
\]

where \( w_{i,t} \) and \( \hat{w}_{i,t} \) are i.i.d not cross-correlated \( N(0,\lambda_i^2) \) and \( N(0,\hat{\lambda}_i^2) \) distributed errors. By restricting \( \rho_i > \hat{\rho}_i \) for every \( i \), this specification allows us to estimate stochastic volatility as a sum of a more permanent and a temporary component. Unlike Everaert and Pozzi (2016), we do not force the first volatility component to follow a random walk process, however later we specify our estimation process to ensure very permanent dynamics. It would be also particularly restrictive to use constant factor loadings because the sovereign credit risk market has experienced several significant shocks, important global and regional crises, that might have generated structural breaks in the dynamics of credit spreads. Thus, we allow each factor loading to evolve according to a random walk:

\[
b_{i,j,t} = b_{i,j,t-1} + v_{i,j,t},
\]

where \( j = \{1, 2, 3, 4\} \) representing the 4 factors, and \( v_{i,j,t} \) are i.i.d. not cross-correlated errors distributed as \( N(0,\eta_{ij}^2) \) respectively. We describe each latent factor as an autoregressive process of order 1 with stochastic volatility:

\[
f_{j,t} = \phi_j f_{j,t-1} + \xi_j^2 e^{\bar{h}_j,t} \hat{u}_{j,t},
\]

where \( \hat{u}_{j,t} \) is the \( N(0,\xi_j^2) \) distributed error term assuming all \( \hat{u}_{j,t} \) innovations are i.i.d. errors, and not correlated with the error terms in equation (7). As we assume no correlation between the error terms of equation (7) and (11), we implicitly state that every co-movement between the credit spreads are due to their exposures to common factors. During crisis periods, the volatility of the common factors can also rise, therefore as induced from equation (11), we allow the factor volatilities to evolve stochastically with the same two-factor specification applied for the individual volatilities, namely:

\[
\begin{align*}
    g_{j,t} &= \psi_j g_{j,t-1} + z_{j,t}, \\
    \hat{g}_{j,t} &= \hat{\psi}_j \hat{g}_{j,t-1} + \hat{z}_{j,t},
\end{align*}
\]

where \( \psi_j > \hat{\psi}_j \) for each \( j \), \( z_{j,t} \) and \( \hat{z}_{j,t} \) are i.i.d not cross-correlated \( N(0,\theta_j^2) \) and \( N(0,\hat{\theta}_j^2) \) distributed errors.
4.3.1 Contagion and interdependence

The main goal of our study is to examine the existence and nature of contagion and interdependence on the global CDS market. Hence, we must consider how these concepts fit into our dynamic factor model. Both effect is strongly related to the importance of the estimated global and regional components. In the factor modeling literature, a natural choice to measure the importance of the components is variance decomposition: for all $t$, the variance of each individual time series can be written as the sum of the variance of each component. In our analysis, this decomposition is formalized as:

$$\text{Var}(y_{i,t}) = \text{Var}(b_{i,G,t}f_{G,t}) + \text{Var}(b_{i,R,t}f_{R,t}) + \text{Var}(\sigma_i^2 e^{h_{i,t}}u_{i,t})$$  \hspace{1cm} (14)

From equation (14), one can compute the proportion of the variation in CDS spread changes explained by the global and regional factors:

$$V_{i,t}^k = \frac{\text{Var}(b_{i,k,t}f_{k,t})}{\text{Var}(b_{i,G,t}f_{G,t}) + \text{Var}(b_{i,R,t}f_{R,t}) + \text{Var}(\sigma_i^2 e^{h_{i,t}}u_{i,t})}$$  \hspace{1cm} (15)

where $k = \{G, R\}$, and $0 \leq V_{i,t}^k \leq 1$. If both $V_{i,t}^G$ and $V_{i,t}^R$ are 0, then country $i$ is completely separated from the global and regional credit market movements. On the other hand, if $V_{i,t}^G$ and $V_{i,t}^R$ adds up to 1, all the CDS spread movement of country $i$ is originated from the global and regional components, and their importance depends on the ratio between $V_{i,t}^G$ and $V_{i,t}^R$. These quantities can be straightforwardly calculated from equation (7)-(13), namely:

$$V_{i,t}^G = \frac{b_{i,G,t}^2(e^{\beta_{G,t}})^2/(1 - \theta_G^2) + b_{i,R,t}^2(e^{\beta_R,t})^2/(1 - \theta_R^2) + \sigma_i^2(e^{h_{i,t}})^2/(1 - \rho_i^2)}{b_{i,G,t}^2(e^{\beta_{G,t}})^2/(1 - \theta_G^2) + b_{i,R,t}^2(e^{\beta_R,t})^2/(1 - \theta_R^2) + \sigma_i^2(e^{h_{i,t}})^2/(1 - \rho_i^2)}$$  \hspace{1cm} (16)

and

$$V_{i,t}^R = \frac{b_{i,R,t}^2(e^{\beta_R,t})^2/(1 - \theta_R^2) + b_{i,R,t}^2(e^{\beta_R,t})^2/(1 - \theta_R^2) + \sigma_i^2(e^{h_{i,t}})^2/(1 - \rho_i^2)}{b_{i,G,t}^2(e^{\beta_{G,t}})^2/(1 - \theta_G^2) + b_{i,R,t}^2(e^{\beta_R,t})^2/(1 - \theta_R^2) + \sigma_i^2(e^{h_{i,t}})^2/(1 - \rho_i^2)}$$  \hspace{1cm} (17)

However, Everaert and Pozzi (2016) argued that using the above-defined measures does not give an adequate measure of market integration, because it suffers from the heteroscedasticity bias. A significant portion of the time variation in $V_{i,t}^G$ and $V_{i,t}^R$ comes from the stochastic nature of the idiosyncratic and factor volatility. During global or regional temporary upheaval periods, the aggregated volatility of the given factor highly increases and indicates more variance share explained by that factor, without having any structural brakes in the market integration and the importance of regional and global factors. Everaert and Pozzi (2016) also propose a
corrected version of the variance ratio measure, by excluding the temporary part of the stochastic volatilities and calculating $V_{G,c}^{i,t}$ and $V_{R,c}^{i,t}$ including only the permanent volatility component, i.e. using $h_{i,t}$ and $g_{k,t}$ instead of $\bar{h}_{i,t}$ and $\bar{g}_{k,t}$:

$$V_{G,c}^{i,t} = \frac{b_{i,G,t}^2(e_{G,t}^{\eta})^2/(1 - \theta_{G}^2)}{b_{i,G,t}^2(e_{G,t}^{\eta})^2/(1 - \theta_{G}^2) + b_{i,R,t}^2(e_{R,t}^{\eta})^2/(1 - \theta_{R}^2) + \sigma_i^2(e_{h_i,t}^2)/(1 - \rho_i^2)}$$

and

$$V_{R,c}^{i,t} = \frac{b_{i,R,t}^2(e_{R,t}^{\eta})^2/(1 - \theta_{R}^2)}{b_{i,G,t}^2(e_{G,t}^{\eta})^2/(1 - \theta_{G}^2) + b_{i,R,t}^2(e_{R,t}^{\eta})^2/(1 - \theta_{R}^2) + \sigma_i^2(e_{h_i,t}^2)/(1 - \rho_i^2)}$$

These variance ratio measures are not affected by the temporary changes of the idiosyncratic and factor volatilities, but captures the effect of the longer-term structural changes. The evolution of $V_{G,c}^{i,t}$ and $V_{R,c}^{i,t}$ depends on the time-varying factor loadings $b_{i,k,t}$, factor volatilities $g_{k,t}$, and idiosyncratic volatilities $h_{i,t}$. A ceteris paribus increase in $b_{i,k,t}$ or $g_{k,t}$ gives more importance to the given factor for country $i$, while if $h_{i,t}$ increases, country $i$ becomes more separated from the global and regional credit risk markets. During our analysis, we use the corrected variance ratio measure $V_{k,c}^{i,t}$ to quantify the market integration and draw inferences about the contagion and interdependence effects. If we find that a significant portion of the variance is permanently explained by the global or regional factors indicated by $V_{k,c}^{i,t}$ for a set of countries, we call it interdependence. On the other hand, if we observe a substantial long-lasting shift in $V_{k,c}^{i,t}$ after a shock occurred or a significant increase in $V_{k,c}^{i,t}$ for a shorter but still relevant interval during a crisis period, we mark it as contagion. However, this model cannot recognize the country who is the source of the shocks but identifies all countries whose linkage is strengthened. Also, if we see a slow increase or decrease in the variance shares, we do not categorize it either contagion or interdependence. Finally, as we estimate global and regional factors, inferences can be made on both levels.

### 4.3.2 Normalization and identification

Several normalization and identification issues are related to the Bayesian factor modeling framework and we must address them. First, we set the variance of the factor innovations $\xi_j$ equal to 1. This normalization is necessary because the factor loadings $b_{i,t}$ and $\xi_j$ cannot be separately identified as one can get two equivalent models by multiplying $b_{i,j,t}$ with $\kappa_j$ and divide the $j$th factor with the same constant. Instead of a constant $\kappa_j$, rescaling $b_{i,j,t}$ and $f_{j,t}$ with a time-dependent $\kappa_{j,t}$ could cause problems, however, due to the autoregressive structure of the factor dynamics, the rescaled values would not satisfy equation (10) and (11) (Del Negro and Otrok, 2008).
The volatility in the innovation term is also subject to normalization problems if the permanent volatility component follows a random walk process, as with an arbitrary constant \( \kappa, \sigma_i^2 e^{2h_{i,t}} = \sigma_i^2 e^{2h_{i,t} + \log(\kappa)} \). Although we do not specify the permanent volatility component as a random walk, to avoid such rescaling issues, we normalize \( h_{i,t} \) over \( t \) in each innovation term by setting \( \sum_{t=1}^{T} h_{i,t} = 0 \).

The temporary part of the stochastic volatility does not suffer from such identifying issues, because it is well identified as a zero-mean AR(1) process. As the factor volatility process is defined in the same way, we also normalize \( g_{j,t} \) over \( t \) in each factor innovation term by setting \( \sum_{t=1}^{T} g_{j,t} = 0 \).

Geweke and Singleton (1980) showed if the combined loading matrix \( B_t = (b_{1,t}', \ldots, b_{M,t}')' \) has a smaller rank than the number of factors, the model is not well identified. This phenomenon is often called the rotation problem and we address this issue by parameters restrictions (Lopes and West, 2004), restricting one country’s credit spread to depend only on the global factor and not on any regional one. In our application we choose this country to be Russia as it has very limited credit risk similarities with any other analyzed countries.

As the Gibbs sampling proceeds, the obtained chain of samples can easily switch sign, because the likelihood is the same with \( \{f_{j,t}, b_{i,j,t}, \ldots\} \) and \( \{-f_{j,t}, -b_{i,j,t}, \ldots\} \). We did not find evidence of this problem during our model runs but just to be sure the factors have the appropriate signs, we redrew the samples for \( b_{i,j,t} \) if their average values are negative at one country for each region, assuming these countries (Germany, Brazil, South Korea) have on average positive loadings on their regional factors.

### 4.3.3 Priors

The estimation of our Bayesian factor model requires prior distribution assumptions for all the innovation variances \( \sigma_i^2, \eta_{i,j}^2, \lambda_i^2, \hat{\lambda}_i^2, \theta_j^2, \text{ and } \hat{\theta}_j^2 \) in equation (7)-(13), the constant terms \( a_i \), the autoregressive coefficients \( \phi_j \) of the factors and for the autoregressive parameters \( \rho_i, \hat{\rho}_i, \psi_j, \text{ and } \hat{\psi}_j \) of the volatility components. For every coefficient type parameters, we assume normal priors \( N(\tilde{\alpha}, \tilde{\beta}) \), where \( \tilde{\alpha} \) shows our expectations on the given coefficient, while \( \tilde{\beta} \) is the prior variance. Addressing the critique arising with the prior selection in the Bayesian approach, we can have all prior expectations being equal to some reasonable value, while the prior variance is set to a larger value, which means we have a less informative prior and the posterior distribution depends mostly on the observed sample. Following this idea, we assume \( N(0, 1) \) normal priors for each \( a_i \) constants. As we are working with differenciated
spreads, we expect the estimated factors to have an autoregressive coefficient close to zero, therefore we assume \( \phi \sim N(0, 0.05^2) \) normal priors for the factor AR1 parameters. In the case of the stochastic volatilities, we expect one of the components to be very close to a random walk process, while the temporary component might have lower, but still large autoregressive coefficient. Treating parameters as random variables, the Bayesian approach incorporates the uncertainty about the parameters, without the necessary of predetermining whether the time-series is stationary or not. Uhlig (1994) shows that assuming normal prior centered around 1 provides a reasonable choice for the autoregressive coefficients of processes with unit-root, hence we assume normal \( N(1, 0.05^2) \) prior for each autoregressive coefficient \( (\rho_i \text{ and } \psi_j) \) related to the permanent part of the stochastic volatilities. Finally, regarding to \( \hat{\rho}_i \text{ and } \hat{\psi}_j \), we also assume a less informative normal prior with lower expectation \( N(0.9, 0.1^2) \).

For every innovation variance, we assume the conjugate inverse gamma prior \( IG(\tilde{\gamma}, \tilde{\delta}) \), where the parameter \( \tilde{\delta} \) shows our expectations on the level of the variance, while larger \( \tilde{\gamma} \) represents more beliefs about the accuracy of \( \tilde{\delta} \). Again addressing the critique arising with the Bayesian approach, we can have \( \tilde{\delta} \) being equal to some reasonable value, while set \( \tilde{\gamma} \) to a small number, which again means we have a less informative prior and the posterior distribution depends mostly on the observed sample. Therefore, we assume \( IG(1, \tilde{\sigma}_i^2) \) inverse gamma prior for each \( \sigma_i^2 \), where \( \tilde{\sigma}_i^2 \) is computed by regressing each individual CDS spread changes with a proxy common factor estimated via principal component analysis, and taking the variance of the residuals. For the other priors related to innovation variances we assume more restrictive priors, namely \( IG(0.05T, 1) \) for \( \eta_i^2 \) and \( IG(0.5T, 0.05^2) \) for all \( \lambda_i^2, \hat{\lambda}_i^2, \theta_j^2 \text{ and } \hat{\theta}_j^2, i = 1, \ldots, M \) and \( j = 1, \ldots, K \). We express \( \tilde{\delta} \) as a function of \( T \), to explicitly incorporate that the magnitude of our belief is related to the number of observations. The priors related to the factor loading and stochastic volatility variances are tighter and concentrated to lower expected variances, demonstrating that we expect the factor loadings and stochastic volatilities to be adjusted slower and smoothly throughout our sample period. In Section 7, along with other robustness questions, we investigate the effect of wider and tighter priors.

All together we have 29067 parameters to be estimated. Samples are drawn 15000 times from the marginal distributions for each parameter. We mark the 5000 realizations as burn-in period samples, thus we discard them. Our empirical results are inferred from the last 10000 realizations. Finally, Table 1 presents the list of parameters along with our prior assumptions and Gibbs sampler initial values \( (T = 168, K = 4, M = 40) \).
\begin{table}
\centering
\begin{tabular}{lcccc}
\hline
Parameter & Count & Prior & Initial value \\
\hline
$a_i$ & 40 & $N(0,1)$ & 0 \\
$\phi_j$ & 4 & $N(0,0.05^2)$ & 0 \\
$\rho_i$ & 40 & $N(1,0.05^2)$ & 1 \\
$\hat{\rho}_i$ & 40 & $N(0.9,0.05^2)$ & 0.9 \\
$\psi_i$ & 4 & $N(1,0.05^2)$ & 1 \\
$\hat{\psi}_i$ & 4 & $N(0.9,0.05^2)$ & 0.9 \\
$\sigma_i^2$ & 40 & $IG(1,\hat{\sigma}_i^2)$ & $\hat{\sigma}_i^2$ \\
$\eta_{ij}^2$ & 30 & $IG(48,0.05^2)$ & 0.005 \\
$\lambda^2_i$ & 40 & $IG(84,0.05^2)$ & 0.05 \\
$\hat{\lambda}^2_i$ & 40 & $IG(84,0.05^2)$ & 0.05 \\
$\theta_i^2$ & 4 & $IG(84,0.05^2)$ & 0.05 \\
$\hat{\theta}_i^2$ & 4 & $IG(84,0.05^2)$ & 0.05 \\
$b_{i,j,t}$ & $30 \times 1$ & $Benchmark$ & model \\
$h_{i,t}$ & $40 \times 1$ & $Factors$ & (Section 6) \\
$g_{i,t}$ & $40 \times 1$ & $Factors$ & (Section 6) \\
$f_{j,t}$ & $4 \times 1$ & Benchmark & model \\
\hline
\end{tabular}
\caption{List of the parameters to be estimated with their priors and initial values}
\end{table}

4.3.4 Estimation

This subsection contains the detailed description of the block Gibbs sampling algorithm we employ to estimate the model parameters. During the whole estimation process of the factor model specified in Section 4.1-4.3, we rely on the methodologies used and described by Kim et al. (1999), Diebold et al. (2008), Del Negro and Otro (2008), and Everaert and Pozzi (2016). Our block Gibbs sampler consists of the following steps, and let $\Omega$ denote all the parameters for which the steps are conditioned:

[1] drawing factor loadings: $b_{i,j,t}|y, \Omega$

For each $i = 1, ..., M$, consider the subset $l = 1, ..., L$ of $j$’s for which $b_{i,j,t}$ is not strictly restricted to be equal to zero, i.e. select the index of those factors that have effect on the $i$th country’s CDS spread. As we assumed independent residuals in equation (10), we can draw the factor loadings for each $i$ separately. Conditioning on all other parameters, the model can be put into a standard state-space form:

\begin{align*}
    y_{i,t}^* &= H_{i,t}X_{i,t} + \epsilon_{i,t} \quad (20) \\
    X_{i,t} &= G_{i}X_{i,t-1} + \tau_{i,t} \quad (21)
\end{align*}

where $\epsilon_{i,t} \sim N(0,R_{i,t}), \tau_{i,t} \sim N(0,Q_{i,t})$, the observed values are $y_{i,t}^* = y_{i,t} - a_i$, and the states are the factor loadings: $X_{i,t} = (b_{i,1,t}, ..., b_{i,L,t})'$. The observation matrix contains
the relevant factors: $H_{i,t} = (f_{1,t}, ..., f_{L,t})$, while the transition matrix represents the independent random walk dynamic of the factor loadings: $G_i = I_L$, where $I_L$ is the $L \times L$ identity matrix. Finally, $R_{i,t} = \sigma_i^2 e^{2h_{i,t} + \hat{h}_{i,t}}$ and $Q_{i,t} = \text{diag}(\eta_{i1}^2, ..., \eta_{iL}^2)$. Next, we use the algorithm of Carter and Kohn (1994), described in Kim et al. (1999) to generate random draws for the level and loading factors. First, we estimate the state-space model with Kalman filter and save the updated estimates $\hat{X}_{i,t|t}$, and their covariance matrices $\hat{P}_{i,t|t}$. The Carter and Kohn (1994) algorithm is a backward recursive procedure with the following steps:

- if $t = T$: draw a random sample $X_{i,T}^*$ from $N(\hat{X}_{i,T|T}, \hat{P}_{i,T|T})$
- for $t = T - 1, ..., 1$, draw a random sample $X_{i,t}^*$ from $N(\hat{X}_{i,t|t}, \hat{P}_{i,t|t})$, where

$$
\begin{align*}
\hat{X}_{i,t|t} &= \hat{X}_{i,t|t} + \hat{P}_{i,t|t} G_i^t [G_i \hat{P}_{i,t|t} G_i^t + Q_{i,t}]^{-1} \hat{X}_{i,t+1|t} - G_i \hat{X}_{i,t|t} \\
\hat{P}_{i,t|t} &= \hat{P}_{i,t|t} - \hat{P}_{i,t|t} G_i^t [G_i \hat{P}_{i,t|t} G_i^t + Q_{i,t}]^{-1} G_i \hat{P}_{i,t|t}
\end{align*}
$$

The sampled factor loadings are simply the corresponding rows of $X_{i,t}^*$.

[2] **drawing factor loading variances:** $\eta_{i,j}^2 | y, \Omega$

Having the same independence assumptions and index sets for each $i$ as above, we can calculate easily calculate $\nu_{i,j}$ residual vectors from equation (10) for $t = 2, ..., T$. As we assume inverse gamma conjugate prior distribution for $\eta_{i,j}^2 \sim IG(\tilde{\eta}_{i,j}, \tilde{\delta}_{i,j})$, we can straightforwardly draw samples from the posterior:

$$\eta_{i,j}^2 | y, \Omega \sim IG\left(\frac{\tilde{\eta}_{i,j}}{2}, \frac{\tilde{\delta}_{i,j}}{2}\right),$$

where $\tilde{\eta}_{i,j} = \tilde{\eta}_{i,j} + T - 1$ and $\tilde{\delta}_{i,j} = \tilde{\eta}_{i,j} \tilde{\delta}_{i,j} + \nu_{i,j}^t \nu_{i,j}$.

[3] **drawing the constant term:** $a_i | y, \Omega$

Conditioned on all other parameters, equation (7) can be rearranged as $y_{i,t}^* = a_i$, where $y_{i,t} = y_{i,t} - b_{i,t} F_t$. This is a standard normal linear regression model with only a constant and known variance, but we have to correct for the stochastic part of the volatility and regressing $y_{i,t}^*$ with $x_{i,t}^* = 1/\epsilon^{2h_{i,t}+2\hat{h}_{i,t}}$. We assumed normal priors for each $a_i, i = 1, ..., M$ as $a_i \sim N(\tilde{a}_{a,i}, \tilde{\beta}_{a,i})$, therefore we can sample from the posterior distribution given as:

$$a_i | y, \Omega \sim N(\tilde{a}_{a,i}, \tilde{\beta}_{a,i}),$$

where $\tilde{\beta}_{a,i} = (\tilde{\beta}_{a,i}^{-1} + \sigma_i^{-2} x_{i}^* x_{i}^*)^{-1}$ and $\tilde{a}_{a,i} = \tilde{\beta}(\tilde{\beta}_{a,i}^{-1} \tilde{a}_{a,i} + \sigma_i^{-2} x_{i}^* y_{i}^*)$.

[4] **drawing the stochastic volatility of equation (7):** $h_{i,t}, \hat{h}_{i,t} | y, \Omega$

As we conditioned on $\Omega$, we can express the residual term of equation (7) as $e_{i,t} = \sigma_i e^{h_{i,t} + \hat{h}_{i,t}|u_{i,t}}$, where $u_{i,t} \sim N(0, 1)$. Taking square first and then log, we get
the following equation:

\[ e_{i,t}^* = \log(\sigma_i^2) + 2h_{i,t} + 2\hat{h}_{i,t} + u_{i,t}^* \]  

where \( e_{i,t}^* = \log(e_{i,t}^2) \), and \( u_{i,t}^* = \log(u_{i,t}^2) \). Considering equations (8)-(9) as state equations, (26) can be put into a state space form, however \( u_{i,t}^* \) is not normally distributed, but follows a \( \log(\chi_i^2) \) distribution. Kim et al. (1998) provides a solution by approximating \( u_{i,t}^* \) with a mixture of normals, formally:

\[ u_{i,t}^* \sim \sum_{\omega=1}^{K^*} q_\omega^* N(m_\omega^* - 1.2704, \nu_\omega^2), \]

or equivalently

\[ u_{i,t}^*|\omega_{i,t}^* = \omega \sim N(m_\omega^* - 1.2704, \nu_\omega^2), P(\omega_{i,t}^* = \omega) = q_\omega^* \]

where parameters \( \{q_\omega^*, m_\omega^*, \nu_\omega^2\} \) are provided in Table 4 in Kim et al. (1998). This involves one more step to our Gibbs sampling algorithm as we have to sample \( \omega_{i,t}^* \), but it is easy as it does not depends on other parameters and discretely distributed with the conditional probability mass function given as:

\[ P(\omega_{i,t}^* = \omega|\Omega) \propto q_\omega^* f_{N(m_\omega^* - 1.2704, \nu_\omega^2)}(e_{i,t}^* - \log(\sigma_i^2) - 2h_{i,t} - 2\hat{h}_{i,t}) \]

where \( f_{N(a,b)} \) is the probability density function of a normal variable with mean \( a \) and variance \( b \). If we sampled \( \omega_{i,t}^* \) for all \( t = 1, \ldots, T \), we can draw samples for \( h_{i,t} \) and \( \hat{h}_{i,t} \) with the Carter and Kohn (1994) algorithm using the state space form for \( y_{i,t} = e_{i,t} - \log(\sigma_i^2) - m_\omega^* + 1.2704 \) with \( X_{i,t} = (h_{i,t}, \hat{h}_{i,t})' \) as states and matrices \( H_{i,t} = (2, 2), R_{i,t} = \nu_\omega^2, G_{i} = diag(\rho_i, \hat{\rho}_i), Q_{i,t} = diag(\lambda_i^2, \hat{\lambda}_i^2) \). Finally, we normalize \( h_{i,t} \) over \( t \) by subtracting the mean \( \frac{1}{T} \sum_{t=1}^{T} h_{i,t} \) from each sampled value.

[5] drawing the volatility of stochastic volatility of equation (7): \( \lambda_i^2, \hat{\lambda}_i^2|y, \Omega \)

As again, conditioned on all other parameters including the stochastic volatilities \( h_{i,t} \) and \( \hat{h}_{i,t} \), we can calculate \( w_i \) and \( \hat{w}_i \) residual vectors from equation (8)-(9) for \( t = 2, \ldots, T \). The sampling method is again just drawing from a posterior inverse gamma distribution, given that we assumed inverse gamma priors \( \lambda_i^2 \sim IG(\tilde{\gamma}_{\lambda,i}, \tilde{\delta}_{\lambda,i}) \) and \( \hat{\lambda}_i^2 \sim IG(\tilde{\gamma}_{\hat{\lambda},i}, \tilde{\delta}_{\hat{\lambda},i}) \):

\[ \lambda_i^2|y, \Omega \sim IG\left(\frac{\tilde{\gamma}_{\lambda,i}}{2}, \frac{\tilde{\delta}_{\lambda,i}}{2}\right), \]

\[ \hat{\lambda}_i^2|y, \Omega \sim IG\left(\frac{\tilde{\gamma}_{\hat{\lambda},i}}{2}, \frac{\tilde{\delta}_{\hat{\lambda},i}}{2}\right), \]

where \( \gamma_{\lambda,i} = \tilde{\gamma}_{\lambda,i} + T, \delta_{\lambda,i} = \tilde{\delta}_{\lambda,i} + w_i \hat{w}_i, \gamma_{\hat{\lambda},i} = \tilde{\gamma}_{\hat{\lambda},i} + T, \) and \( \delta_{\hat{\lambda},i} = \tilde{\delta}_{\hat{\lambda},i} + w_i \hat{w}_i. \)
[6] drawing the AR(1) coefficient of the stochastic volatility: \( \rho_i, \hat{\rho}_i | y, \Omega \)

We assumed normal priors in equation (8) and (9) for each \( \rho_i \) and \( \hat{\rho}_i \), as \( \rho_i \sim N(\bar{\alpha}_{\rho,i}, \bar{\beta}_{\rho,i}) \) and \( \hat{\rho}_i \sim N(\hat{\alpha}_{\rho,i}, \hat{\beta}_{\rho,i}) \). For \( t = 2, \ldots, T \), it is a standard normal linear regression model with known variance: \( y_i^* = x_i^* \rho \), where \( y^* \) contains the known stochastic volatilities for \( t = 1, \ldots, T \) and \( x^* \) is their lagged value vector for both \( h_{i,t} \) and \( \hat{h}_{i,t} \). Therefore, we can sample from the posterior distribution given as:

\[
\rho_i | y, \Omega \sim N(\bar{\alpha}_{\rho,i}, \bar{\beta}_{\rho,i}), \quad \hat{\rho}_i | y, \Omega \sim N(\hat{\alpha}_{\rho,i}, \hat{\beta}_{\rho,i}),
\]

where \( \bar{\beta}_{\rho,i} = (\bar{\beta}_{\rho,i}^{-1} + \lambda_i^{-2} x_i^* x_i^*)^{-1}, \bar{\alpha}_{\rho,i} = \bar{\beta}(\bar{\beta}_{\rho,i}^{-1} + \lambda_i^{-2} x_i^* y_i^*), \bar{\beta}_{\hat{\rho},i} = (\bar{\beta}_{\rho,i}^{-1} + \lambda_i^{-2} x_i^* y_i^*) \) and \( \hat{\alpha}_{\rho,i} = \hat{\beta}(\hat{\beta}_{\rho,i}^{-1} + \lambda_i^{-2} x_i^* y_i^*). \)

Finally, we redraw samples if \( \hat{\rho}_i \) is larger than \( \rho_i \) to ensure that the first component captures the long term permanent dynamics of the stochastic volatility.

[7] drawing the non-time-varying volatility of equation (7): \( \sigma_i^2 | y, \Omega \)

Conditioning on all other parameters, we can easily calculate the residuals from equation (7) for each \( i \) separately, but now we also have to correct for the stochastic volatility part to get appropriate random samples for \( \sigma_i^2 \). Let \( e_i^2 \) denotes the corrected residual vector for \( t = 1, \ldots, T \), where \( e_i^2 = (y_{i,t} - a_{i,t} - b_{i,t} F_t) / e_{h_{i,t} + \hat{h}_{i,t}} \). We assume inverse gamma priors for \( \sigma_i^2 \sim IG(\hat{\gamma}_{\sigma,i}, \hat{\delta}_{\sigma,i}) \) and draw posterior samples similarly as in the case of the loading variances from:

\[
\sigma_i^2 | y, \Omega \sim IG\left(\frac{\hat{\gamma}_{\sigma,i}}{2}, \frac{\hat{\delta}_{\sigma,i}}{2}\right),
\]

where \( \hat{\gamma}_{\sigma,i} = \gamma_{\sigma,i} + T \), and \( \hat{\delta}_{\sigma,i} = \delta_{\sigma,i} + e_i^2 \).

[8] drawing the stochastic volatility of the factors: \( g_{j,t}, \hat{g}_{j,t} | y, \Omega \)

Similarly as for the individual stochastic volatilities, we conditioning on \( \Omega \) and express the residual term of equation (11) as \( e_{j,t} = e^{g_{j,t} + \hat{g}_{j,t}} u_{j,t} \), where \( u_{j,t} \sim N(0, 1) \).

Again, taking square first and then log, we get the following equation:

\[
e_{j,t}^* = 2g_{j,t} + 2\hat{g}_{j,t} + \hat{u}_{j,t}^*
\]

where \( e_{j,t}^* = \log(e_{j,t}^2) \), and \( \hat{u}_{j,t}^* = \log(\hat{u}_{j,t}^2) \). Considering equations (12)-(13) as state equations, (35) can be put into a state space form and the same sampling scheme is applied as for the individual stochastic volatilities. The conditional probability mass function of \( \omega_{j,t}^* \) is now given as:

\[
P(\omega_{j,t}^* = \omega | \Omega) \propto q_\omega \int N(m_\omega^* - 1.2704, \nu_\omega^* 2)(e_{j,t}^* - 2g_{j,t} - 2\hat{g}_{j,t})
\]

\[\text{(36)}\]
If we sampled $\omega_{j,t}^*$ for all $t = 1, \ldots, T$, we can draw samples for $g_{j,t}$ and $\hat{g}_{j,t}$ with the Carter and Kohn (1994) algorithm using the state space form for $y_{j,t}^* = e_{j,t}^* - \nu_{\omega}^2 + 1.2704$ with $X_{j,t} = (g_{j,t}, \hat{g}_{j,t})'$ as states and matrices $H_{j,t} = (2, 2)$, $R_{j,t} = \nu_{\omega}^2$, $G_j = diag(\psi_j, \hat{\psi}_j)$, $Q_{i,t} = diag(\theta_j^2, \hat{\theta}_j^2)$. Finally, we normalize $g_{j,t}$ over $t$ by subtracting the mean $\frac{1}{T} \sum_{t=1}^{T} g_{j,t}$ from each sampled value.

[9] drawing the volatility of stochastic volatility of the factors: $\theta_j^2, \hat{\theta}_j^2 | y, \Omega$

As again, conditioned on all other parameters including the stochastic volatilities $g_{j,t}$ and $\hat{g}_{j,t}$, we can calculate $z_j$ and $\hat{z}_j$ residual vectors from equation (12)-(13) for $t = 2, \ldots, T$. The sampling method is again just drawing from a posterior inverse gamma distribution, given that we assumed inverse gamma priors $\theta_j^2 \sim IG(\tilde{\gamma}_{\theta,j}, \tilde{\delta}_{\theta,j})$ and $\hat{\theta}_j^2 \sim IG(\tilde{\gamma}_{\hat{\theta},j}, \tilde{\delta}_{\hat{\theta},j})$:

$$\theta_j^2 | y, \Omega \sim IG\left(\frac{\tilde{\gamma}_{\theta,j}}{2}, \frac{\tilde{\delta}_{\theta,j}}{2}\right),$$

$$\hat{\theta}_j^2 | y, \Omega \sim IG\left(\frac{\tilde{\gamma}_{\hat{\theta},j}}{2}, \frac{\tilde{\delta}_{\hat{\theta},j}}{2}\right),$$

where $\tilde{\gamma}_{\theta,j} = \tilde{\gamma}_{\theta,j} + T$, $\tilde{\delta}_{\theta,j} = \tilde{\delta}_{\theta,j} + \tilde{z}_j z_i$, $\tilde{\gamma}_{\hat{\theta},j} = \tilde{\gamma}_{\hat{\theta},j} + T$, and $\tilde{\delta}_{\hat{\theta},j} = \tilde{\delta}_{\hat{\theta},j} + \tilde{z}_j z_i$.

[10] drawing the AR(1) coefficient of the stochastic volatility of the factors: $\psi_j, \hat{\psi}_j | y, \Omega$

We assumed normal priors in equation (12) and (13) for each $\psi_j$ and $\hat{\psi}_j$, $j = 1, \ldots, K$, as $\psi_j \sim N(\tilde{\alpha}_{\psi,j}, \tilde{\beta}_{\psi,j})$ and $\hat{\psi}_j \sim N(\tilde{\alpha}_{\hat{\psi},j}, \tilde{\beta}_{\hat{\psi},j})$. For $t = 2, \ldots, T$, it is a standard normal linear regression model with known variance: $y_t^* = x_t^* \psi$, where $y^*$ contains the known factor stochastic volatilities for $t = 1, \ldots, T$ and $x^*$ is their lagged value vector for both $g_{j,t}$ and $\hat{g}_{j,t}$. Therefore, we can sample from the posterior distribution given as:

$$\psi_j | y, \Omega \sim N(\tilde{\alpha}_{\psi,j}, \tilde{\beta}_{\psi,j}),$$

$$\hat{\psi}_j | y, \Omega \sim N(\tilde{\alpha}_{\hat{\psi},j}, \tilde{\beta}_{\hat{\psi},j}),$$

where $\tilde{\beta}_{\psi,j} = (\tilde{\beta}_{\psi,j}^{-1} + \theta_j^{-2} x_j^t x_j^*)^{-1}$, $\tilde{\alpha}_{\psi,j} = \tilde{\beta}(\tilde{\beta}_{\psi,j}^{-1} \tilde{\alpha}_{\psi,j} + \theta_j^{-2} x_j^t y_j^*)$, $\tilde{\beta}_{\hat{\psi},j} = (\tilde{\beta}_{\hat{\psi},j}^{-1} + \hat{\theta}_j^{-2} x_j^t x_j^*)^{-1}$ and $\tilde{\alpha}_{\hat{\psi},j} = \tilde{\beta}(\tilde{\beta}_{\hat{\psi},j}^{-1} \tilde{\alpha}_{\hat{\psi},j} + \hat{\theta}_j^{-2} x_j^t y_j^*)$. Finally, we redraw samples if $\hat{\psi}_j$ is larger than $\psi_j$ to ensure that the first component captures the long term permanent dynamics of the stochastic volatility.

[11] drawing the AR(1) coefficient of the factors: $\phi_j | y, \Omega$

We assumed normal priors in equation (11) for each $\phi_j$, $j = 1, \ldots, K$, as $\phi \sim N(\tilde{\alpha}_j, \tilde{\beta}_j)$. For $t = 2, \ldots, T$, it is a standard normal linear regression model with known variance, but we also have to correct for the stochastic part of the factor volatility and regressing $y_{j,t}^*$ with $x_{j,t}^* = f_{j,t}/e^{2\theta_j t + 2\hat{\theta}_j t}$ as $y_t^* = x_t^* \phi$, where $y^*$ contains
the known factor values for \( t = 2, \ldots, T \) and \( x^* \) is their lagged volatility corrected values. Therefore, we can sample from the posterior distribution given as:

\[
\phi_j | y, \Omega \sim N(\bar{\alpha}_{\phi,j}, \bar{\beta}_{\phi,j}),
\]

where \( \bar{\beta}_{\phi,j} = (\bar{\beta}_{\phi,j}^{-1} + x_j^* x_j^* - 1)^{-1} \) and \( \bar{\alpha}_{\phi,j} = \bar{\beta}_{\phi,j}^{-1} \bar{\alpha}_{\phi,j} + x_j^* y_j^* \).

[12] drawing the latent factors: \( f_{j,t} | y, \Omega \)

Finally, we draw samples from the latent factors utilizing the Carter and Kohn (1994) algorithm. Let \( y_t = (y_{1,t} - a_{1,t}, \ldots, y_{M,t} - a_{M,t})' \), \( B_t = (b_{1,t}', \ldots, b_{M,t}')' \), and \( \Phi = diag(\phi_1, ..., \phi_K) \). Using these notations, equation (7) and (11) can be expressed in state-space form:

\[
y_t = B_t F_t + \epsilon_t
\]

\[
F_t = \Phi F_{t-1} + \tau_t,
\]

where \( \epsilon_t \sim N(0, R_t) \) and \( \tau_t \sim N(0, Q_t) \). Now, we can jointly draw samples from the latent factors using the method described in the first Gibbs sampling step with: \( X_t = F_t, H_t = B_t, G = \Phi, R_t = diag(\sigma_1^2 e^{2h_{1,t}}, \ldots, \sigma_M^2 e^{2h_{M,t}}) \), and \( Q_t = diag(e^{2g_1,t}, \ldots, e^{2g_K,t}) \).

5 The Data

Our database consists of 5-year sovereign credit spreads of 40 countries covering emerging and developed economies from 3 geographical regions: Europe, Latin America, Southeast Asia between 2005 January and 2018 December. The observations are monthly closing values obtained from Bloomberg Terminal. The 5-year credit defaults swaps are generally the most liquid among all maturities that justifies our choice, while the timespan was chosen based on data availability. The list of covered countries contains 22 European: Germany, France, Belgium, Netherlands, Austria, Finland, Ireland, Italy, Portugal, Spain, Greece, Estonia, Slovakia, United Kingdom, Sweden, Norway, Hungary, Poland, Romania, Bulgaria, Turkey, Russia; 8 Latin American: Brazil, Chile, Colombia, Mexico, Peru, Venezuela, Argentina, Costa Rica; and 10 South-East Asian economies: South Korea, Philippines, Indonesia, Japan, China, Malaysia, Thailand, Vietnam, Hong Kong, Australia. Alternatively, we could have built a model on sovereign bonds, but the CDS market is more liquid, reacts faster to short-run changes and credit spreads are more exact measures of a sovereign’s default related risks.

Unfortunately, the obtained dataset is not flawless, as we have missing values, while for some countries, the spreads are not available from 2015. To estimate
the factor model, we need data points for every time period, thus we linearly interpolated between the available observations and set the spreads equal to their first observed value until the first data point is available for each country (3.8% of all datapoints). We also normalized every time series by dividing all spread values by its country-level standard deviation, thus $y_{i,t}$ denotes the normalized CDS spreads. This normalization is necessary to avoid problems caused by outlier countries such as Greece and Venezuela. Country-level descriptive statistics are presented in Table 2 in the Appendix for the individual CDS spread levels and changes. Among the covered countries, Norway has the lowest mean (19.7) during our sample period, while the largest mean spread is linked to Venezuela (2185.8), followed by Greece (1577.2) who also has the largest observed value (25423). For most of the countries, the average spread changes are near to 0, but due to the large increases, the average spread changes for Greece, Venezuela and Argentina are 2-digit positive numbers, and these countries also show the highest spread change standard deviations. Figure 1 presents credit default swap spreads for 8 countries analyzed in this study. Note that Greece and Venezuela are on the second axis and dots indicate interpolated values. It is easily seen how the credit spread of these countries increased during the financial crisis, while the European credit spreads reached a higher level during the sovereign debt crisis.

![Credit default swap spread of 8 analyzed countries](image)

Figure 1: Credit default swap spread of 8 analyzed countries (Greece and Venezuela on secondary axis)
We also performed a standard principal component analysis on a global and regional level as well. Globally the first and second component explains 55% and 64% of the variation in the CDS spread changes, while the first component for only European, Latin American, and Southeast Asian countries explains 60%, 59%, and 71% respectively. These results are in line with the previous findings, however, we estimated slightly less commonality among the Latin American countries, which contradicts to Hibbert and Pavlova (2017). Figure 2 shows the first and second global principal component and the second component obtained from the PCA applied only for European CDS spreads. We can see that most of the countries have roughly similar weights on the first global component, however, countries such as Greece, Venezuela, and Argentina who has experienced country level crisis periods during our sample have lower loadings. The second global component can be easily associated with the difference between European (blue) and non-European countries (green), which is the same finding as in Longstaff et al. (2011). The first components estimated on regional levels mostly give equal weights for each country, but the second European principal component indicates the possible difference between Eurozone members (blue) and non-member states (orange) as presented in Figure 2.
6 Empirical results

Before we turn to our final results, we also set up a benchmark model to test how the outcome of our model relates to simpler specifications. In the benchmark model, we estimate a proxy global factor simply as a first principal component obtained from principal component analysis including all countries in Section 5. Then we control for this global factor in each time series by regressing the spread differences with the estimated global component. To get proxies for each regional factor, we apply a second principal component analysis using the regression residuals (as a part not explained by the global factor) and set the regional factors equal to the obtained first component.

6.1. Estimated factors

Figure 3 presents the estimated global and regional factors from our Bayesian dynamic factor model as posterior means obtained from the Carter and Kohn (1994) algorithm for the latent factors. The shaded grey area represents the 95% highest probability intervals, which is a similar (but not interchangeable) concept in Bayesian econometrics to the frequentist confidence intervals. We find that before 2008, there is no significant global and European component on the sovereign credit risk market, but our model was able to precisely estimate the global and European factors after 2008 as the significant differences from the zero line and the narrow highest probability intervals indicate. After the two financial crisis, the components show less variation, but we can still measure significant values. The Latin American and Southeast Asian factor shows less variation during the whole sample and the estimated values are often not significantly different from zero. However, a less significant factor does not mean that it does not explains fair amount of the CDS spread variations as it has to be evaluated along with the other components.

Figure 4 shows the cumulated posterior means of the estimated factors (blue lines), along with the components from the benchmark model (orange lines) rescaled to have the same mean and variance as our estimated time series. The shaded grey area represents the 95% highest probability interval of the spread changes. As expected, the global component starts to boost after 2007 and later peeks in the second half of 2008 and during 2009. Then it returns to lower levels but rises again around 2012 indicating a strong worldwide co-movement during the European sovereign crisis. On the other hand, the European factor also begins to continuously increase after the financial crisis until it peeks in 2012 and 2013. As for the Latin American
factor, we see a significant decrease in the CDS spreads, which is associated with the recovery process of the Latin American countries after a recession period in the early 2000s. Again connected to bad real economic performance and political issues, their spreads also increased in 2015-2016. The Southeast Asian factor shows a slight increase during our sample period, but as we have seen in Figure 3, these changes are less significant. Comparing the cumulated estimated factors with their proxy counterparts, we find they have similar dynamics, but our factors have more clear economical interpretations and evolve more realistically than the proxy as they are also controlled to time-variant dynamics.

6.2. Evidence of contagion and interdependence

Analyzing the factors is important to get an overall picture of the co-movements in the sovereign credit spread market, but the contagion and interdependence effects

Figure 3: Posterior means of the dynamic factors with 95% highest probability intervals
Figure 4: Posterior means of the cumulated dynamic factors with 95% highest probability interval of the spread changes and benchmark factors can be inferred from the dynamic of the corrected variance measures $V_{i,t}^{G,c}$ and $V_{i,t}^{R,c}$ indicating changes in the market integration on a global and regional level. As stated above, interdependence means the time-varying $V_{i,t}^{G,c}$ and $V_{i,t}^{R,c}$ have relatively stable and significant values during the whole sample, while any significant change or substantial shift is categorized as contagion. First, we investigate these phenomena on a global level, presented in Figure 5. The solid colored lines are posterior means of the global variance ratios $V_{i,t}^{G,c}$ obtained from equation (18), along with their 90% highest probability intervals as gray shaded areas. We also present the posterior means for the not corrected variance ratios $V_{i,t}^{G}$ (solid black line with the dotted lines indicating the 90% highest probability intervals) to highlight the importance of excluding temporary volatility changes. The dash-dotted vertical black line indicates the first reliable observed CDS spread change for each country where the values were extrapolated. In order to keep the transparency of the time series for 40 countries, we carefully structured these figures by coloring each line according to their regional locations (blue, green, dark red), while we also highlighting the PIIGS countries (red). We also grouped the countries with similar economic characteristics and estimated results;
from top rows to the bottom: core countries, PIIGS, mostly Northern European sovereigns, not Eurozone members, Latin American economies and Southeast Asian countries, while note that the first 13 countries are member of the Eurozone. As we estimated a huge number of parameters from a medium-sized sample, it is not surprising we have wider highest probability intervals, but we can still infer from the dynamics of the posterior means.

Figure 5: Posterior means of $V_{i,t}^{G,c}$ and $V_{i,t}^G$ with 90% highest probability intervals

Looking at the results we find that most of the $V_{i,t}^{G,c}$ have relatively stable significant values during the whole sample, but some differences among countries are also presented. The global market integration of the Eurozone member states (first 13 countries) is on average low, the global factor explains only 20-30% of the total variation measured by $V_{i,t}^{G,c}$, and the average values show less fluctuation. For some countries (Belgium, Austria), $V_{i,t}^{G,c}$ slightly increases during the 2008 financial crisis,
while until adopting the Euro in 2011, the Estonian CDS spreads increasingly depend on the global factor. On the other hand, the global market integration for France, Belgium, Italy, Portugal, and Spain is shifted downward as the financial crisis has begun, signaling a decrease in the connectedness of these sovereigns to the global CDS market. For the non-Eurozone countries, we find more connectedness to the global factors, especially in the case of the Eastern European countries, where about the 50% of the total variation is explained by the global component.

Turning to the Latin American and Asian countries we find the global market integration is also higher than for the Eurozone states, but there is no overall clear pattern regarding the evolution of $V_{G,c}^{G,e}$. Some countries have on average slowly increasing variance ratios (Vietnam, Indonesia), while others show more rapid linkage growth during the financial crisis (Brazil, Colombia, Peru, South Korea, Philippines) indicating possible contagion, and others have relatively stable values with slight decreases after the turmoil periods (Chile, Mexico). Finally, for three Latin American countries (Venezuela, Argentina, Costa Rica), who experienced country-level economic and political crises, furthermore for Japan and Hong Kong, whose economic dynamics traditionally differs from the rest of the world, the global credit risk integration is much smaller, possibly because their credit risk movements are determined by country-level shocks.

Figure 6 and 7 demonstrate our estimations for the variance ratios on regional factors $V_{R,c}^{R,e}$. The solid colored lines are posterior means obtained from equation (19), along with their 90% highest probability intervals as gray shaded areas (for the sake of transparency, Turkey is not presented, but its $V_{R,c}^{R,e}$ posterior distribution is centered around the zero line as Turkey is less connected to the European credit market). We also present the posterior means for the not corrected variance ratios $V_{G}^{G,e}$ (solid black line with the dotted lines indicating the 90% highest probability intervals). Based on the market integration evolutions, several Eurozone member states experienced strong contagion during our sample period. On average, the regional variance ratio of all core European and PIIGS countries (excluding Greece) has increased, but for France, Belgium, Ireland, Italy, Portugal, and Spain, the sudden shift in the credit market integration indicating strong contagion. On the other hand, the non-member countries show less exposure to the European regional factor.
Regarding the Latin American countries, we estimate low market integration for Venezuela, Argentina, and Costa Rica on a regional level too, but the other countries have a significant portion of variation explained by the estimated Latin American component, although the exposure decreased during the global crisis periods. For
the Southeast Asian countries, we cannot find a common pattern, as one group of countries have significant exposure to the regional, while others are mostly affected by the global and idiosyncratic factors, however, we couldn’t find any fundamentals that might clearly separate these two groups. Considering the non-corrected variance ratios (black solid line) in Figure 5-7, we find substation deviations from the corrected market integration measure, which validates the choice of the correction proposed by Everaert and Pozzi (2016) on the sovereign credit risk market. As an example, the $V_{G,t}^G$ and $V_{R,t}^R$ tends to underestimate the market integration before the financial crises, and overestimates the increase in the variance ratios, indicating a false contagion effect, contaminated by the heteroskedasticity bias even for countries, where we did not find relevant changes during the sample period.

In order to draw accurate inference about the contagion and interdependence effects based on the presented market integration dynamics, we have to analyze $V_{G,t}^{G,c}$ parallel to $V_{R,t}^{R,c}$ and to the variation not explained by the global and regional factors. Figure 8 presents a stacked area plot containing the posterior means for both variance ratios during the sample interval. The dark grey area represents the proportion of CDS spread change explained by the global factor, while the colored (blue, green, red) areas show the market integration on a regional level. As all the subplots are aggregated to 1, the residual area indicates the effect of the idiosyncratic part and how the given country is separated from the global and regional credit risk market.

We find that most of the markets are highly integrated into the global and regional markets, which indicates strong interdependence on both global and regional levels, although we cannot state the CDS market is growingly integrated as a large part of the variation is still originated from the idiosyncratic components at the end of the sample period. The level and the evolution of the linkages show differences and a few subgroups can be considered. Sovereigns, such as Greece, Venezuela, Argentina, who experienced local economic and political crises show very low interdependence with the common credit risk markets, while the reason for the low values for Japan is probably due to its traditional economic difference from the other countries. We also found evidence of contagion at several sovereigns on a global level after the 2008 financial crises, but the variance ratios tend to decrease after the turmoil periods.

For the Latin American and Asian sovereigns, the proportion of the aggregated variance explained by the global and regional factors fluctuates around a high value, therefore the increase in the global exposure is followed by the decrease in the regional connectedness. Investigating the corrected variance ratios of the first twelve presented countries, we find the change of the market integration composition moved
to the other direction: the importance of the regional component sharply increased after the financial crisis and stayed higher during the European sovereign crisis and throughout the rest of the covered years. This observation is clear evidence of contagion among these countries. However, as for the other European countries (from Estonia to Russia), the integration into the European market is much smaller. We find, these differences can be traced back to one condition: whether the given country is a member of the Eurozone since 2009 or not, which shows how shocks have changed the credit risk connectedness and the sensitivity to regional fundamentals in the Eurozone after 2009, but not in all Europe. We also present the average values of the posterior means of $V_{i,t}^{G,c}$ and $V_{i,t}^{R,c}$ for 4 groups of countries: Eurozone members, non-Eurozone European countries, Latin American, and Southeast Asian sovereigns in Figure 9, supporting our results regarding the global and Eurozone level contagion.
Figure 9: Mean of the corrected variances explained by the global and regional factors

6.3. Sources of contagion

Section 4.3.1 showed that the applied market integration measure $V_{G,c}^{t}$ and $V_{R,c}^{t}$ relies on three time-varying components, namely on the factor loadings $b_{i,k,t}$, factor volatilities $g_{k,t}$, and idiosyncratic volatilities $h_{i,t}$. In this section, we investigate the evolution of each component to understand what changes caused the observed global and European contagion effects.

Figure 10 shows the permanent $e^{g_{j,t}}$ (blue) and aggregated $e^{g_{j,t} + \hat{g}_{j,t}}$ (black) stochastic volatilities of the 4 estimated factors along with their 90% percent highest probability intervals (gray shaded area and the area between the dotted lines). As the permanent log-volatility is normalized to have an average equal to 0 and the temporary part is modeled as a zero mean autoregressive process, if the stochastic volatility assumption is not valid, the estimated time series should not deviate from a constant line with the value equal to 1. However, we find that the aggregated stochastic volatility of the global and European factors increased significantly during the financial turmoil in 2008-2009 and stayed on higher levels until the markets calmed down after the European sovereign crisis. The change is also significant if we consider only the permanent component, therefore we can identify the structural change in the long-run stochastic volatility between 2008 and 2014 as a possible source of the observed contagion. The Latin American factor volatility decreases before the financial crisis, but the permanent component does not significantly different from 1 but still decreasing. Finally, the Southeast Asian factor volatility has the least time-variation and it is not considered to be as any source of contagion.
Figure 10: Posterior means of the permanent and aggregated stochastic volatility of the factors with 90% highest probability intervals

The structural breaks in the factor volatility can indicate deeper market integration but the increased idiosyncratic volatility has the opposite effect and decreases the importance of the common factors. Figure 11 presents the permanent $e^{h_{j,t}}$ (blue) and aggregated $e^{h_{j,t} + \hat{h}_{j,t}}$ (black) idiosyncratic stochastic volatilities of the 40 sovereigns along with their 90% percent highest probability intervals (gray shaded area and the area between the dotted lines). Again, the permanent log-volatility is normalized in the estimation process to have an average equal to 0 and the temporary part is modeled as a zero mean autoregressive process. Figure 11 shows that after controlling for changes in the volatility at a global and regional level, we can still estimate significant variation in the idiosyncratic part of the CDS spread changes, which influences the size of the market integration measures. We see evident similarities across the European countries as their individual volatility peaks between 2008 and 2013. However, the difference again exists between Eurozone members and non-members, as the volatility in the former group usually reaches its maximum in 2012 during the European debt crisis, while for the latter group of countries, it peaks
mostly at the financial turmoil in 2008-2009 or at both events. The findings in the
other two geographical regions are more mixed, but some country related events can
be recognised such as the effect of the political and economic crisis in Venezuela.

![Graph showing data for various countries](image)

Figure 11: Posterior means of the permanent and aggregated idiosyncratic stochastic
volatility with 90% highest probability intervals

The final source of the time variation in \( V_{G,c}^{i,t} \) and \( V_{R,c}^{i,t} \) is the evolution of the
global and regional factor loadings. These coefficients show how the exposure to
the common factors have changed for country \( i \) (but not the variation explained by
latent factors). First, we investigate the loading changes on a global level, presented
in Figure 12. The solid lines are posterior means of global factor loadings obtained
from the Carter and Kohn (1994) algorithm, along with their 90% highest probability
intervals as gray shaded areas. Looking at the results we find that most of the
loadings have relatively stable positive values during the whole sample, but some differences among countries are also presented. The global factor loadings for Italy, Portugal, and Spain are shifted upward as the European debt crisis has begun, signaling a possible source of contagion experienced by these states, while loadings for Ireland and Greece stayed low because their credit risk fundamentals are locally determined. As for the other European countries, we find their global factor loadings are mostly fluctuating around a significant positive value, except for Austria and Estonia, where the increased market integration is driven by the growth in their global factor loadings. On the other hand, the global factor loadings of the Latin American and Asian countries are more stable in time and evolved more smoothly.

Figure 12: Posterior means of the global loadings with 90% highest probability intervals
Figure 13 and 14 demonstrate our estimations for the loading on regional factors. The solid lines are posterior means obtained from the Carter and Kohn (1994) algorithm, along with their 90% highest probability intervals as gray shaded areas. We again find relatively stable and slowly evolving loadings for the Latin American and Asian sovereigns, therefore we claim the contagion experienced by the countries in these regions is driven by the structural change in the permanent volatility of the global factor. However, Figure 13 provides us more interesting results regarding the European regional factor exposures (for the sake of transparency, Turkey is not presented, but its loading posterior distribution is centered around the zero line as Turkey is less connected to the European credit market). The sensitivities to the regional component sharply rise right after the global financial turmoil for the core European and PIIGS sovereigns (except for Greece) and this significant change also increases the calculated regional market integration measures. Investigating the loadings of the rest of the countries, we do not find any similar reaction to the financial crisis, posterior mean values are fluctuating around some positive values with minor changes.

Figure 13: Posterior means of the regional loadings for European countries with 90% highest probability intervals
Figure 14: Posterior means of the regional loadings for Latin American and Southeast Asian countries with 90% highest probability intervals

To summarize our finding, we claim that as the global and regional factor loadings for the non-Eurozone members, the Latin American countries and the Asian sovereigns mostly fluctuates around a positive value with low variation, or evolves slowly during the sample period, and the regional factor volatilities are relatively stable, the contagion experienced by these countries is driven by the structural change in the volatility of the global common factor. The regional contagion in the Eurozone countries originated from the permanent increase in the European factor volatility and the sharp rises in the regional factor loadings, even though the structural shift in the global and idiosyncratic volatilities tend to lower the regional market integration.

7 Robustness tests

We also tested the robustness of our results in several dimensions: (1) we changed the prior distribution for the factor loading variations to $IG(168, 0.001^2)$, which means the factor loadings are forced to have a very small variation and we are obtaining estimates with nearly constant loadings; (2) we also dropped the stochastic part of the volatility from the model, by setting $h_{i,t}$, $\hat{h}_{i,t}$, $g_{j,t}$, and $\hat{g}_{j,t}$ equal to 0 for all $i$, $j$ and $t$; (3) we dropped only the factor stochastic volatility from the model, by setting $g_{j,t}$ and $\hat{g}_{j,t}$ equal to 0 for all $j$ and $t$. We also estimated models separately with wider
priors: (4) IG(33, 0.005^2) for the factor loading; (5) N(1, 0.1^2) and N(0.9, 0.5^2) for
the autoregressive coefficients in the permanent and temporary part of the stochastic
volatility for both the factors and the idiosyncratic part; (6) IG(33, 0.05^2) for all
the innovation variances in the permanent and temporary part of the stochastic
volatility for both the factors and the idiosyncratic part. We expect (1)-(3) to have a
significant effect on our results as they act as the exclusion of time-dependency from
our model, while (4)-(6) shouldn’t affect our inference qualitatively.

Figure 15: Average market integration measure differences from the original results

We estimated \( V_{G,c}^{G,c} \) and \( V_{R,c}^{R,c} \) for all new model specifications and calculated
their averages for 4 groups of countries as in Figure 9. Figure 15 presents the
difference between the averaged market integration time series from each robustness
test model and our base result. As expected, we find that using less informative
priors for the factor loading innovations and the volatility of stochastic volatility
does not change the overall finding of our analysis. On the other hand, restricting
the models by diminishing the time-variation effect in the factor loadings and the
volatility has an effect on the estimated variance ratios, which would also bias our
inference about the contagion and interdependence. These findings justify our model specification, showing that including time-dependent dynamics results with more realistic estimations without biases caused by restricting the given variables.

8 Future research

The analysis in this thesis can be expanded in several directions in the future. The sovereign credit spreads can be decomposed into default risk and risk premium components. The former represents jump to default risk i.e. the risk of the credit event occurrence, while the latter incorporates the risk that the intensity parameter corresponding to the probability of the default event changes in time. The decomposition could be carried out with the model of Remolona et al. (2007) who decomposed the CDS spreads via the rating announcements of Standard and Poor’s and Moody’s or by the method described in Pan and Singleton (2008) and used by Longstaff et al. (2011) extracting information from the term structure of CDS spreads. It would be an interesting analysis to investigate how the factor structure and connectedness of these two parts behave and what differences can be observed.

Augustin (2018) also incorporates the information content available from the CDS spread term structure and shows how the difference between the 10-year and 1-year spreads, defined as slope influences the global and country-specific risk in explaining the CDS spread variation. We analyzed the spreads of 5-year maturity CDS contracts, however, the same estimation process can be carried out for any other maturities if the data points are available. Our hypothesis states that for the longer maturities, a larger proportion of the spread variation is explained by global and regional factors, whereas the short-end spreads are heavily influenced by country-level shocks. Using the term structure model of Diebold and Li (2006), we could also compute a slope and curvature time series for each sovereign and the co-movement of these quantities could be investigated. Finally, as a feedback loop of the risk transformation between the sovereigns and their banks is observed (Dieckmann and Plank, 2011), including bank CDS spreads to the model would also give an additional layer of our analysis.

9 Conclusion

This thesis presented our analysis of the contagion and interdependence effects on the global and regional credit default swap markets. Building on the existing literature, we propose an advanced model to investigate the existence and nature of these two effects using 14-year data of 40 countries. Our model is a Bayesian Dynamic
Factor model with time-varying parameters, global and regional specific factors, and two-component stochastic volatility. The factor structure captures the co-movement among the country’s CDS spreads without explicitly predetermine common economic fundamentals, while allowing the parameters to adjust in time reflects the fact, that the connectedness structure of the CDS market might have changed during the last 14 years as two major crises were experienced. Finally, the stochastic volatility of the factors and the individual part of the sovereign spreads incorporate the effect of the increased variation during common and individual crisis periods. We apply a time-varying variance decomposition method to quantify the market integration on global and regional levels, however, the simple variance ratio is contaminated by the heteroscedasticity bias, as in the case of the correlation measures, during temporary high volatility periods, it tends to overestimate the real connectedness of financial markets. Therefore, we utilize a corrected version of the variance decomposition proposed by Everaert and Pozzi (2016) to draw inference about the interdependence and contagion effects on the CDS market.

In line with the existing literature, we observe a high degree of commonality in the CDS market, but we detect more significant latent factors on global and European level, while the Latin American and Asian latent factors show less variation during the sample period. Our factors also have clear economic interpretation as their cumulated version show the common spread movements during the global financial turmoil and the sovereign debt crisis. We find evidence of interdependence on both global and regional levels, but the contagion effect is also present for some country groups. Although there is no common pattern across the 40 sovereigns, the integration into the global market has quickly increased for some countries (e.g.: Estonia, Brazil, Colombia, Peru, South Korea, Philippines) after the global financial crisis. We also detect a difference between the CDS spread dynamics of the Eurozone member and non-member countries in the European region. While the latter group of countries is more integrated into the global CDS market, clear evidence of strong contagion is present among the Eurozone member states, as their linkage to the common European factor is sharply increased after the financial crisis, indicating a shift from global integration to a more specific regional integration. The source of this contagion is originated from the increased long-run volatility of the European factor and the shift in the factor loadings, whereas the global level contagion is mostly driven by the permanent rise of the global factor volatility during the financial and sovereign debt crisis. Overall, our analysis fits into the existing literature and highlights the properties of the market integration evolution in sovereign credit default swap market.
References


Appendix

The following table lists all the analyzed countries with their descriptive statistics:

<table>
<thead>
<tr>
<th>Country name</th>
<th>Num. obs</th>
<th>Level</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Min</td>
</tr>
<tr>
<td>Germany</td>
<td>165</td>
<td>26,3</td>
<td>0,9</td>
</tr>
<tr>
<td>France</td>
<td>158</td>
<td>49,4</td>
<td>1,4</td>
</tr>
<tr>
<td>Belgium</td>
<td>168</td>
<td>60,3</td>
<td>1,3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>155</td>
<td>33,3</td>
<td>1,1</td>
</tr>
<tr>
<td>Austria</td>
<td>165</td>
<td>44,2</td>
<td>1,5</td>
</tr>
<tr>
<td>Ireland</td>
<td>131</td>
<td>194,2</td>
<td>19,6</td>
</tr>
<tr>
<td>Italy</td>
<td>168</td>
<td>144,8</td>
<td>3,5</td>
</tr>
<tr>
<td>Portugal</td>
<td>168</td>
<td>243,9</td>
<td>2,5</td>
</tr>
<tr>
<td>Spain</td>
<td>168</td>
<td>124,4</td>
<td>1,9</td>
</tr>
<tr>
<td>Greece</td>
<td>165</td>
<td>1577,2</td>
<td>4,0</td>
</tr>
<tr>
<td>Slovakia</td>
<td>168</td>
<td>67,1</td>
<td>2,0</td>
</tr>
<tr>
<td>Finland</td>
<td>151</td>
<td>25,3</td>
<td>1,4</td>
</tr>
<tr>
<td>Estonia</td>
<td>146</td>
<td>105,7</td>
<td>5,5</td>
</tr>
<tr>
<td>Sweden</td>
<td>144</td>
<td>26,4</td>
<td>1,6</td>
</tr>
<tr>
<td>Norway</td>
<td>131</td>
<td>19,7</td>
<td>4,9</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>141</td>
<td>40,7</td>
<td>1,5</td>
</tr>
<tr>
<td>Hungary</td>
<td>168</td>
<td>193,4</td>
<td>10,0</td>
</tr>
<tr>
<td>Poland</td>
<td>168</td>
<td>89,2</td>
<td>8,0</td>
</tr>
<tr>
<td>Romania</td>
<td>168</td>
<td>179,1</td>
<td>17,5</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>168</td>
<td>158,3</td>
<td>13,0</td>
</tr>
<tr>
<td>Turkey</td>
<td>168</td>
<td>225,9</td>
<td>117,8</td>
</tr>
<tr>
<td>Russia</td>
<td>166</td>
<td>193,1</td>
<td>38,7</td>
</tr>
<tr>
<td>Brazil</td>
<td>168</td>
<td>196,0</td>
<td>62,2</td>
</tr>
<tr>
<td>Chile</td>
<td>168</td>
<td>74,5</td>
<td>13,2</td>
</tr>
<tr>
<td>Colombia</td>
<td>168</td>
<td>160,2</td>
<td>77,4</td>
</tr>
<tr>
<td>Mexico</td>
<td>168</td>
<td>121,5</td>
<td>28,6</td>
</tr>
<tr>
<td>Peru</td>
<td>168</td>
<td>137,9</td>
<td>62,2</td>
</tr>
<tr>
<td>Venezuela</td>
<td>163</td>
<td>2185,8</td>
<td>122,0</td>
</tr>
<tr>
<td>Argentina</td>
<td>146</td>
<td>1137,2</td>
<td>193,2</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>125</td>
<td>238,4</td>
<td>85,0</td>
</tr>
<tr>
<td>South Korea</td>
<td>164</td>
<td>80,3</td>
<td>14,3</td>
</tr>
<tr>
<td>Philippines</td>
<td>165</td>
<td>163,9</td>
<td>59,2</td>
</tr>
<tr>
<td>Indonesia</td>
<td>160</td>
<td>195,3</td>
<td>81,4</td>
</tr>
<tr>
<td>Japan</td>
<td>161</td>
<td>44,9</td>
<td>1,0</td>
</tr>
<tr>
<td>China</td>
<td>166</td>
<td>76,5</td>
<td>10,0</td>
</tr>
<tr>
<td>Malaysia</td>
<td>165</td>
<td>95,7</td>
<td>12,9</td>
</tr>
<tr>
<td>Thailand</td>
<td>161</td>
<td>96,3</td>
<td>26,9</td>
</tr>
<tr>
<td>Vietnam</td>
<td>164</td>
<td>225,1</td>
<td>56,0</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>159</td>
<td>43,0</td>
<td>4,5</td>
</tr>
<tr>
<td>Australia</td>
<td>98</td>
<td>47,2</td>
<td>13,2</td>
</tr>
</tbody>
</table>

Table 2: Country level descriptive statistics