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★**Connections in combinatorial optimization.**
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The title of the book is wisely chosen: it deals, among other subjects, with graph connectivity, and it provides connections between graph theoretical results and underlying combinatorial structures. The graph theoretical results span orientations, flows, paths, arborescences, dijoins, connectivity augmentation, and many more. Most have been chosen for their connection to polymatroids and submodular flows. That is why matching results for other than bipartite graphs have been left out, and why stable sets do not take much space. The book is readable for students, researchers, possibly also practitioners. It may serve as a survey of the above-mentioned areas, a starting point for a researcher's own work, or a tool case for proofs and algorithms, together with a framework for sorting the tools. It contains contributions of Mader, Edmonds, Lovász, Király, and a number of other Hungarian mathematicians, Nagamochi, Ibaraki and other Japanese graph theoreticians, and, of course, András Frank's work.

Part 1 describes basic results and basic proof techniques used later, such as network optimization, matroid theory and polyhedral combinatorics, with a number of questions that point toward possible generalizations. These are mostly answered in Part 2, which covers in depth orientations, packing and covering with trees and arborescences, algorithms for flows and cuts, and connectivity augmentations. These results are proved with graph theoretical techniques, such as splitting-off edges from nodes, reorientation of paths, and uncrossing of sets. Should one get distracted by the abundance of results and variations, one may skip ahead to Part 3, which describes the underlying structures, namely matroid optimization, generalized polymatroids, submodular flows, and covering supermodular functions by digraphs. This part sews everything neatly together in a conclusion in a manner similar to the conclusion of a good criminal novel.

The practitioner may want to take a closer look at the inequalities describing the problem. If the problem at hand contains inequalities constraining the in-degree or the difference of in- and out-degree of node sets by sub- or supermodular functions or a weaker form thereof, a relaxation of the problem at hand can probably be solved polynomially. Also separation of inequalities may be possible, though this is not specifically pointed out. Generic algorithms for optimization and feasibility check for submodular flows and matroid intersection are given. Problems where the number of induced edges of each node set is bounded from above by a sum of node values plus/minus a constant are described in the section on count matroids.

All in all, the book can be recommended to anybody dealing with these topics.

Reviewed by *Mechthild Opperud*